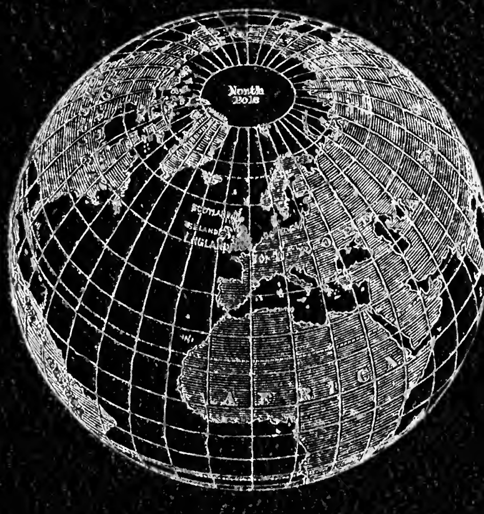
# MAPS

## THEIR USES AND CONSTRUCTION

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## THEIR USES AND CONSTRUCTION

A SHORT POPULAR TREATISE ON THE ADVANTAGES AND DEFECTS OF MAPS ON VARIOUS PROJECTION FOLLOWED BY AN OUTLINE OF THE PRINCIPLES INVOLVED IN THEIR CONSTRUCTION


BY

abriel

G. JAMES MORRISON, 1841-19

MEMB. INST. C.E., F.R.G.S.

## ERRATA

Page 31, 9th line from bottom—for Panama read Bermuda.

Page 60, 5th line from bottom—after "down" insert "as."

Page 87, 16th line—for 92 read 52.

Page 96, 10th line—for "Spheroidal" read "spherical."

Page 104, 13th line from bottom—for 40° read 44°.

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# MAPS

## THEIR USES AND CONSTRUCTION

## CHAPTER I

### INTRODUCTORY

EVERYONE from time to time makes use of maps, but hardly anyone ever consults a globe, and it is not going too far to say that in the case of ninety-nine people out of a hundred all their ideas of the relative size, shape, and positions of the various countries in the world are derived entirely from maps, and generally from those on Mercator's projection; yet of these people probably not one in ten knows that the ideas so derived are inaccurate to a degree which may fairly be called misleading.

On Mercator's maps, for instance, the size of all polar countries is enormously exaggerated. Greenland appears to be many times the size of India, whereas it has approximately the same area; as regards position, the direct route from, say, London to Shanghai appears to pass through the Caspian Sea, while as a matter of fact it passes north of St. Petersburg. In these and various other ways not only maps on Mercator's

projection but all maps of large areas are and must be faulty.

It is proposed in the following pages, in the first place to explain in a popular way why it is that no absolutely correct map can be made, and what are the good and bad points of maps constructed on various systems ; and to follow this up with a general explanation of the mathematical bases of the most useful projections, for the benefit of those interested in this branch of the subject.

It is hardly necessary to state that no difficulties of any magnitude are encountered in making maps of moderate areas. A flat map of England may be made, which, for the purpose of studying geography, may be looked upon as absolutely accurate ; but a map showing half the world or the whole world must be distorted to a considerable extent. While, however, this is true, it is also true that in such maps it is generally possible to preserve some one important quality by sacrificing others. Thus (1) a map may be made on which all small portions of the world retain very nearly their proper shapes, but countries near the edges of the map are on a larger scale than those near the centre. Or (2) a map may be made on which all countries retain their areas accurately, but the shape of each is somewhat distorted. Again (3) a map may be made on which all points retain the proper compass bearings of the routes between them ; that is to say, if on the map one port is north-east of another port, then a ship starting from the latter and sailing north-east will reach the former, but the course will not be the shortest possible : a state-

ment which will be made clear hereafter. Or (4) a map may be made on which a straight line drawn from one point to another will indicate accurately the shortest route on the surface of the globe; but in both these maps, (3) and (4), the distortion near the edges is very great.

The first-named map is very commonly used for giving people general ideas; that is to say, for teaching geography. The second is useful for giving good ideas as to relative size, and as the distortion of shape is not very great it deserves to be in general use, while, as a matter of fact, it is hardly known. The third, or Mercator map, is useful as a sailing chart for moderate voyages, and has the advantage of showing the entire inhabited world on one sheet, a quality which has led to its being used for general purposes to an extent which is quite unjustified. The fourth is useful for laying down long ocean routes, but as it is not on the whole a convenient sailing map it is much less used than it ought to be.

It may seem strange to begin a treatise on maps by what may appear to be a general condemnation of the entire species. The fact is that maps of the world, or of large portions of the world, are at best miserable make-shifts; but as they are absolutely necessary for many purposes, it has been thought it would be useful if the principles on which they are founded could be explained in a manner that would be clear to those who have not made a study of mathematics, so that they might see for themselves how far any particular map might be relied on, and how far people must be on their guard against using a map for a purpose for which it is not

suited, and so being misled, and badly misled, by appearances.

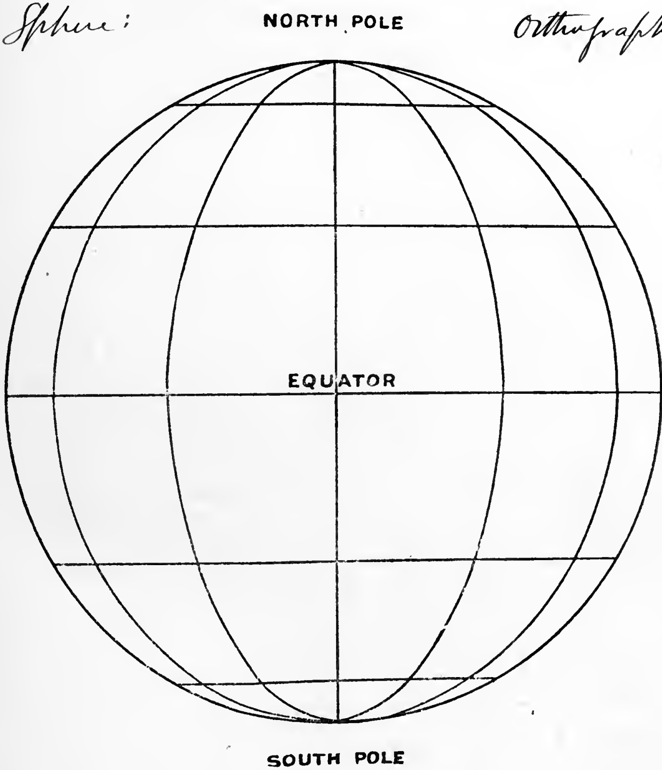
As everyone knows, the earth is a sphere, and though it is slightly flattened at the poles it will be assumed in this portion of the treatise, as is usual in general treatises on projections, that it is a perfect sphere. An accurate model of the earth may therefore be made in the form of a globe, but the inconvenience of using globes is so great that people are practically driven to flat maps. Now, if a correct representation of the world can be made on a globe, the simplest way in principle to make a flat map would be to lay a piece of tracing paper on the globe and trace the map through. A single attempt will show the futility of this method. Unless a very small piece of paper be used, it will not lie flat. Of course no piece, however small, will lie absolutely flat, but a comparatively small piece will lie so close to the surface of the globe that no appreciable crumpling takes place, and the portion of the globe covered can be traced so as to give a flat map without appreciable error. The larger the globe the larger the piece of paper that can be used, but the larger also will be the countries, so that the area to be thus shown cannot be increased. A map of the British Isles might perhaps with difficulty be accomplished, but if one tries to cover half the globe one finds the paper crumple up so as to make tracing impossible. Now this difficulty lies at the root of all map-making, and it cannot be abolished; but there are certain methods by which, to a limited extent, it can be evaded.

The positions of places on the earth are identified by

latitude and longitude. Practically all educated people know what is meant by these words, and some readers may feel inclined to skip the next few pages, but it is advisable to explain how the latitude and longitude of

FIG. 1

Sphere: Orthographic p




places are determined in order to make the succeeding pages of this treatise more easily understood. The surface of a sphere begins and ends nowhere. Any point on it bears the same relation to the whole that

every other point bears, and there is no reason to choose one point more than another to start from. In the case of the earth, however (see fig. 1), there is something. It revolves on its axis once a day, and this axis is therefore a definite line, the ends of which are called poles, one being called North and the other South. Here, then, is something definite to go upon. The earth is supposed to be divided into two equal parts by a line which goes round the world, half-way between the poles, and is called the Equator. The position of this, being defined with reference to the poles, is likewise definite. All circles which divide a sphere into two equal parts are called great circles, and the Equator is therefore a great circle. It has for a long time been the habit to divide a circle into 360 equal parts, called degrees. There is no reason why 360 should have been chosen rather than 400 or 100 or any other number, but it has been used so long that one has come to look upon it as a natural division. 90 degrees therefore means one fourth of a circle, 45 degrees means one eighth, 60 degrees means one sixth, and so on, a degree being indicated by a small circle after the figure, thus: 60°.

Having decided to divide the Equator into 360 degrees, one is met by the difficulty of having no point at which to begin. It will be better to leave this question for a moment and return to it.

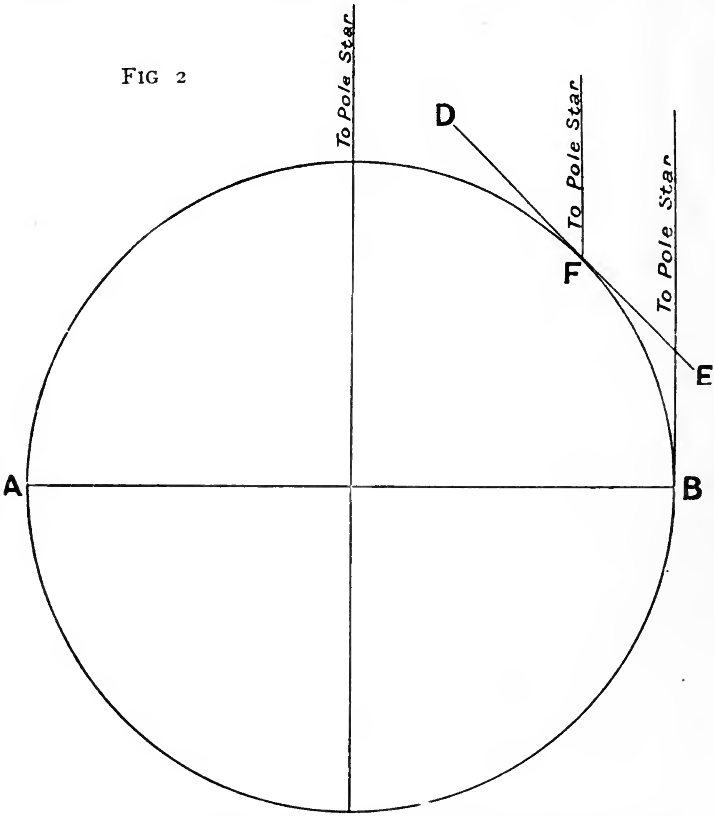
It is clear that any number of great circles can be drawn through the two poles, each of which will cut the Equator into two equal parts. Each of these may be divided into 360°, and in this case the Equator is taken as the starting-point, and there will be 90° between the

Equator and each pole on each side. It is usual to number these from the Equator towards the pole. Now suppose some point is chosen measuring 10° north of the Equator on one of these great circles (which, as will be seen later on, are called meridians) and a circle is drawn through it parallel to the Equator, this circle will be the parallel of 10° north latitude and will cut every meridian 10° north of the Equator. In the same way other circles may be drawn at 20°, 30°, &c., to represent parallels of 20° and 30° of latitude, and further circles may be drawn at every 10° or at every degree north and south, and all of these will be parallels of latitude. It is clear that each of these parallels of latitude cuts the sphere, not into two equal parts, but into two unequal parts. The parallels of latitude are therefore not great circles, but small circles. The point to be established, however, is that the Equator has been drawn with reference to the poles and the parallels have been drawn with reference to the Equator, and are therefore definite in position. There is absolutely nothing arbitrary except the numbering of the degrees. The parallel which lies half-way between the Equator and the pole is called the parallel of 45°. It might be called by another name, but none the less it is a definite line on the surface of the globe, and not an arbitrary line.

The latitude of any place is determined on the following principles. There happens to be a star very nearly in the prolongation of the earth's axis to the north, to which the name pole-star is given. Suppose anyone to be at the North Pole, this star would appear to him to be directly overhead. Suppose again a person

to be at the Equator, the star would appear to him to be on the horizon, level with his eye. It might be said that it would appear below his eye, because it is in line with the axis of the earth, 4,000 miles below his feet, but the

FIG 2




distance of the star is so enormous that this is nothing. All lines to it from any point on the earth appear to be parallel. Suppose a person to be at F (see fig. 2), half-way between the Equator and the pole, the inclined

line on the earth, D E, will appear to him to be horizontal, and he will see the star half-way up the heavens. Now this is equivalent to saying that the latitude of any place is indicated by the height of the pole-star. Most people who have travelled much have observed as they go south that the pole-star night by night appears lower in the heavens and gradually disappears while the Southern Cross gradually comes into view. It is true that at sea the latitude is fixed every day at noon by an observation of the sun, but this is because the sun is brighter and more easily observed. Its distance from the pole, which varies throughout the year, is tabulated for each day in a book called the Nautical Almanack, and when it is observed its polar distance is allowed for, and what is actually stated as the latitude of the ship is in fact the height of the pole in the heavens, and the star itself is as a matter of fact directly observed from time to time. This shows that latitude is not arbitrary. If the star appears one fourth of the way up, measured from the horizon towards the zenith, then the point of observation is one fourth of the way up from the Equator towards the pole, and nothing can alter this. It is of course to be understood that when the pole-star is spoken of, the pole (the prolongation of the earth's axis), to which the star is close, is really meant. When the star itself is observed the difference is allowed for. Southern latitudes are based on the same principles.

Strictly speaking, this is what is called the astronomical latitude of a place. There are, as will be shown later, other latitudes which slightly differ from the above, partly because the earth is not a true sphere and

partly from local attractions, but the above described latitude is not only the one adopted in all general treatises, it is the one adopted in all general maps and charts, whether issued by the Admiralty or by others, and is the latitude by which all navigation is conducted; and, assuming the earth to be a perfectly uniform sphere, it is the only latitude.

This, however, fixes only the parallel of latitude on which the point of observation is situated. If it be found that the latitude of one place is 10° north and the latitude of another 20° north, then the second place is 10° north of the former, but there is nothing to show whether it is east or west of it. Suppose at any point on the earth's surface a perpendicular pole be erected. The shadow of the pole in the morning will be on the west side, and in the evening on the east side, and there is a certain moment when it will lie due north and south. That moment is called noon, and it will be the same for all points exactly north or south of the point of observation. A great circle passing through the poles of the earth and through the point of observation is called a meridian (from *meridies*, mid-day), and it is therefore noon at the same moment at all points on that meridian. Suppose a chronometer keeping correct time to be set to noon and then carried to some other point of observation and noon observed at the new point and compared with the chronometer, and suppose it is found that at noon of the new point the chronometer marks two o'clock, then the meridian on which the new point is situated is one-twelfth of the way round the world to the westward of the first point. This difference is

absolute and definite, there is nothing arbitrary about it, but it would be most inconvenient to have to work simply with differences of various places, and all would be chaos unless some one place was agreed upon as the starting-point. This question cropped up as soon as longitudes began to be thought of and long before they were accurately fixed, and a great many places in turn were used; but as soon as English people began to make charts they adopted the meridian that passed through their principal observatory of Greenwich as the standard from which all longitudes should be measured, and this meridian has now been adopted by many other countries; France, however, takes the meridian of Paris. The adoption of one meridian as standard rather than another is purely arbitrary. The divisions of the Equator are made to begin where the standard meridian crosses it, and degrees are counted 180° each way. The great circle, therefore, which passes through Greenwich is called the meridian of Greenwich or meridian 0° on one side of the globe, and the 180th meridian on the other side, it being 180° east and also 180° west of the zero meridian. By setting a chronometer to Greenwich time and observing the hour of noon at various places their longitude can be determined, allowing 15° to each hour of time, because the earth turns once on its axis in 24 hours and there are 360° in the entire circumference. It is of course to be understood that this is only a rough outline of the principles on which longitude is determined. The exact determination of longitude is a work of considerable difficulty, and the longitudes of the principal observatories have

not even at the present day been fixed with that degree of accuracy which is necessary for certain delicate observations.

It is clear that if a globe have the lines of latitude and longitude drawn on it on the principles above stated, and the latitude and longitude of a certain number of places be determined by observation, these places can be plotted in their proper positions on the globe and the detail can be filled in by ordinary surveying. If in the same way lines to represent latitude and longitude be drawn on a piece of paper, the places can be plotted with reference to these lines and the detail filled in by surveying as before; and the art of making maps consists in the first place in laying down the lines to represent latitude and longitude, either as nearly like the lines on the globe as is possible in transferring lines from a curved to a flat surface, or else in such a way that some one property of the lines will be retained at the expense of others. To transfer the irregular coast-lines from a globe to a map would be a work of super-human difficulty; but to transfer regular lines is comparatively easy, and it is of course possible to lay down on a map lines corresponding to those which would be drawn on an imaginary globe many feet or even many miles in diameter. The lines of latitude and longitude may be laid down for every 10°, or every degree, or every quarter degree, or less if necessary; but the principle is—lay down the lines, plot the principal points, and fill in by surveying. After one map has been made it may of course be copied even on to another projection, care being taken that the latitude and longitude

of every point are kept correct throughout. It is clear that if the lines of latitude and longitude cannot be accurately laid down on a flat surface, still less can the detail be laid down. This difficulty is got over as follows:—For the first maps such a moderate area is taken that there is no practical difference between the flat surface and the curved surface, and it will be assumed that the maps so made are bounded by lines of latitude and longitude.[1] When a large number of these maps have been made it will be found that they cannot be joined together to lie flat.

If they are carefully joined along the edges it will be found that they naturally adapt themselves to the shape of a globe; therefore another piece of paper is taken and on it are laid down lines of latitude and longitude, and each map is *copied* so as to fill the place prepared for it.

Sometimes this can be done by simple reduction which does not affect accuracy, as the accuracy of a map is independent of the scale, but sometimes the correct map has to be enlarged in one direction and reduced in another, and sometimes it has to be twisted, and in almost every case it has to be manipulated to get it into its place. The work of making maps of large areas, therefore, consists of two parts: first, making correct maps of small areas, which may be called surveying; and secondly, laying down lines of latitude and longitude into which these maps must be fitted, and this second part is called map projection.

[1] The arrangement by which maps may be made of rectangular shape to join up flat over a limited area such as an English county will be dealt with in the second part of this treatise.

# CHAPTER II

## MAP PROJECTION CONSIDERED POPULARLY

THE art of map projection, as stated above, consists in laying down lines of latitude and longitude on a flat piece of paper, either in such a way that correct maps of the various spaces can be filled in with as little distortion as possible, or else so that some one good property may be retained at the expense of others which are sacrificed. The map projector's work is therefore finished when these lines are laid down, but in the diagrams in this part of the treatise an outline of the continents will generally be added, so as to make the diagrams clearer to the ordinary reader.

The first system of projection which naturally suggests itself is to make a picture of the globe. When one looks at a globe one sees a little less than half, but if the globe is at a great distance one may assume without appreciable error that one sees half. Figs. 3 and 4 show two views of the earth on this projection, which is called ORTHOGRAPHIC. It will be seen in each case that while the centre of the map is good the edges are very much crowded, and the result is so unsatisfactory that one at once asks if it is not possible to compress the central portion into a smaller area, and so leave a little

more space for the outer portions. The projection which is now to be explained meets this difficulty to some extent.

The principle on which it is founded is shown in

Equat. hemisphere FIG. 3 Orthographic pro

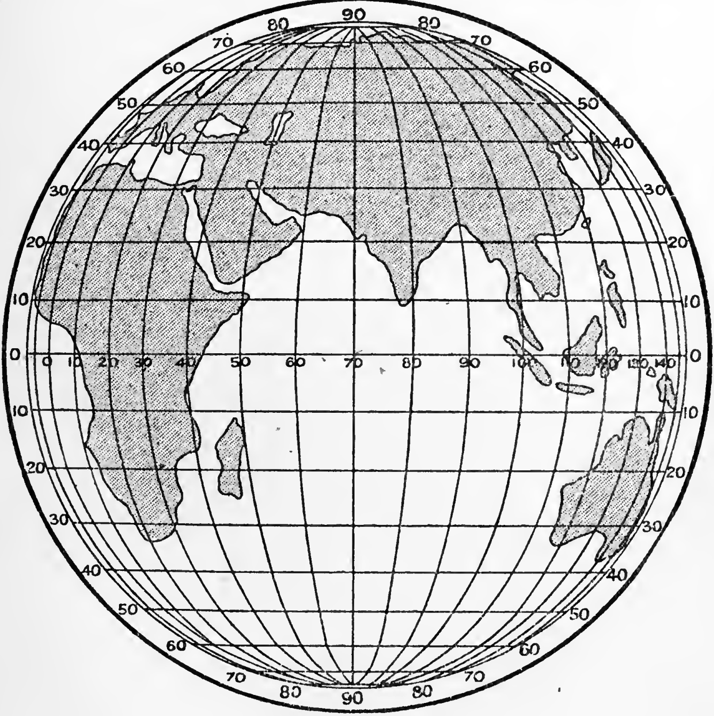
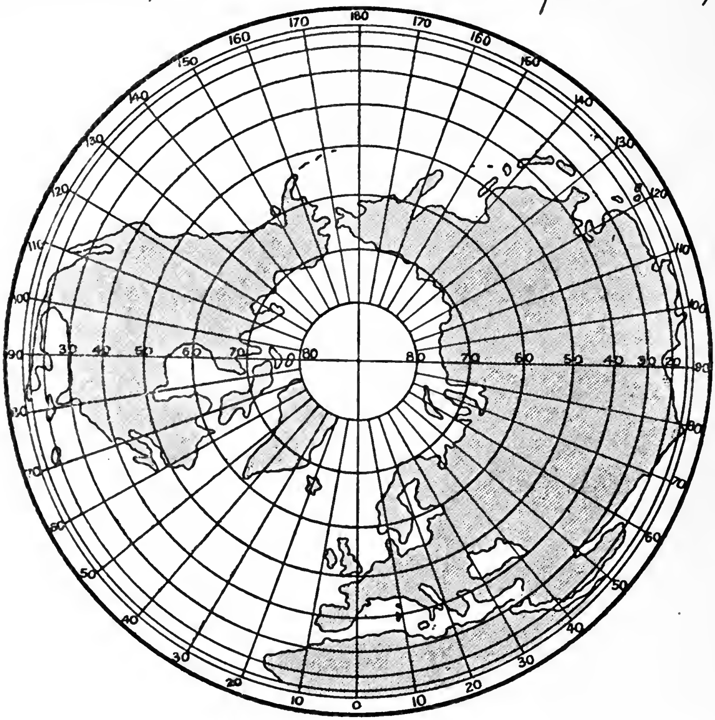



fig. 5. Suppose a globe to be cut in two at A B, and a piece of paper placed between the two halves. Then suppose straight rods like knitting-needles to be run through from any points, F and H, to the point E, passing

through the paper at the points G and K. By using a sufficient number of points the lines of latitude and longitude can be marked out on the paper.

Three specimens of this projection, which is called STEREOGRAPHIC, are shown in figs. 6, 7, 8. The first of

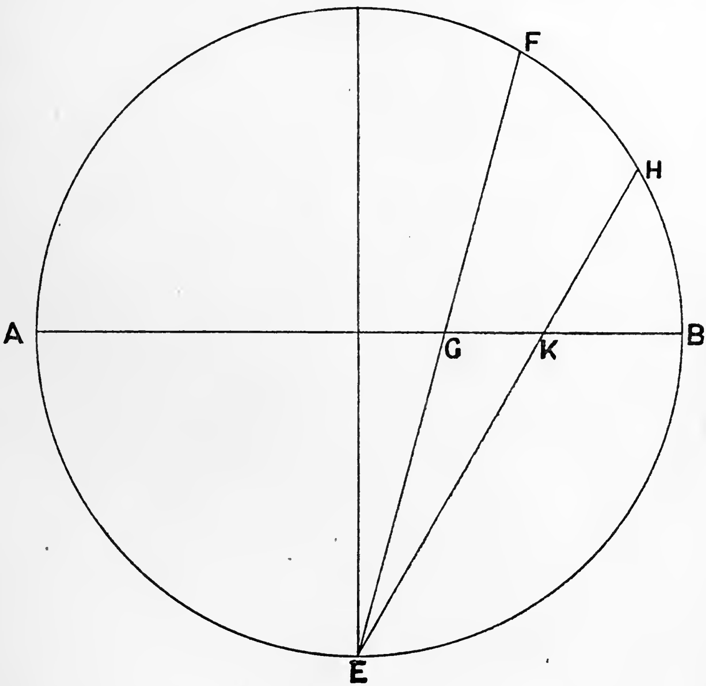
Polar hemisphere. FIG. 4 Orthographic proj.




these is the Northern Hemisphere, the second what is usually called the Eastern Hemisphere, and the third the hemisphere the central point of which is the intersection of the parallel of 50 with the meridian of

Greenwich (a point, that is to say, close to London). On the diagrams lines are drawn for each 10° or 20° of latitude and longitude, but it is clear that on a larger map lines could be drawn for every degree. Now a practically correct map can be made of a portion of the

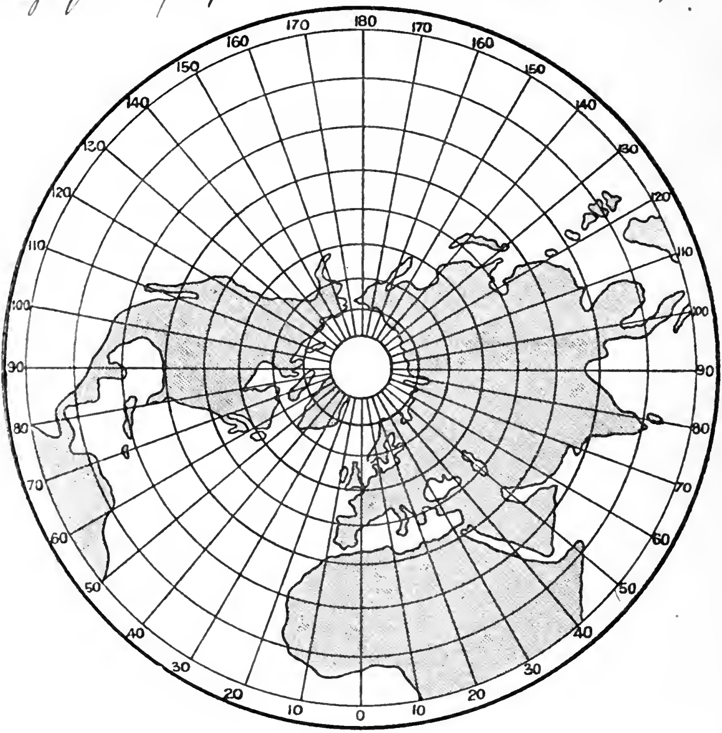
FIG. 5 Principle of Stereop




world measuring 1° each way, because curvature in such a size is of no consequence. Suppose, then, that correct maps were made separately of all the little four-sided portions, it would be found that by simply reducing each of them to the requisite scale it could be fitted

almost exactly into the space to which it belonged. The words 'almost exactly' are used because the edge nearest the centre of the map would have to be compressed a little, but so little that only very careful measurement would detect the difference.

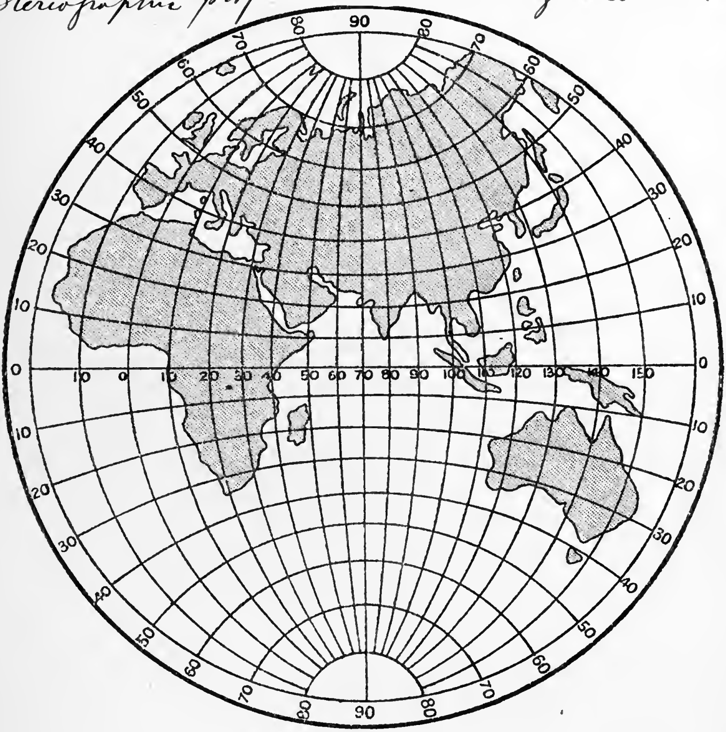
Stereographic Proj. FIG. 6 Polar hemisph.




Here, then, at first sight, it appears that a perfect projection has been obtained, and as a matter of fact it is considered by most authorities to be the best projection of a hemisphere for general purposes, but it has

one serious defect. It has been said that each plan has to be compressed at its inner edge, and for the same reason each plan in succession has to be reduced to a smaller average scale than the one outside of it. In

Stereographic proj' FIG. 7 Equatorial Hem

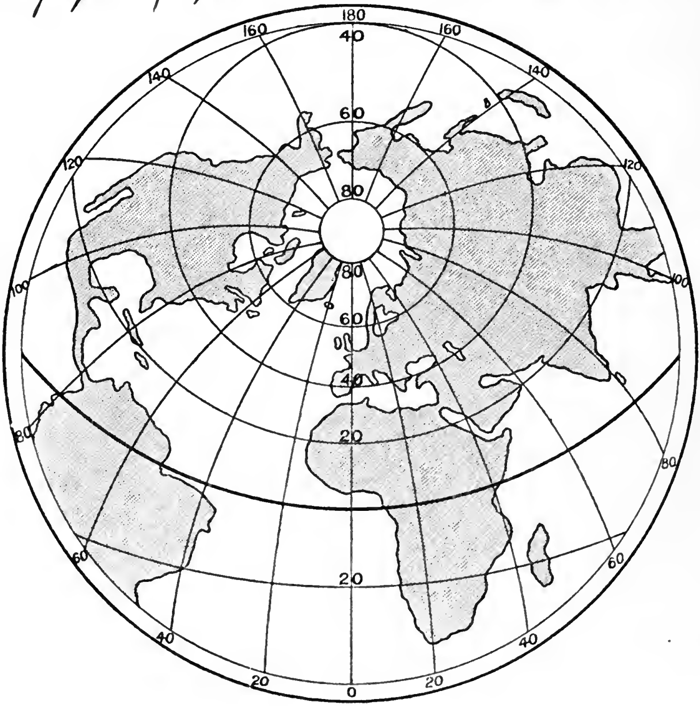



other words, the *shape* of each space into which a plan has to be fitted is practically correct, but the *size* is less in proportion at the centre than at the edges, so that if a correct plan of an area at the edge of the map has to be reduced to a scale of 500 miles to an inch to

c 2

fit its allotted space, then a plan of an area at the centre has to be reduced to a scale of 1000 miles to an inch. Thus a moderate area has its true shape, and even an area as large as India is not distorted to an extent to be

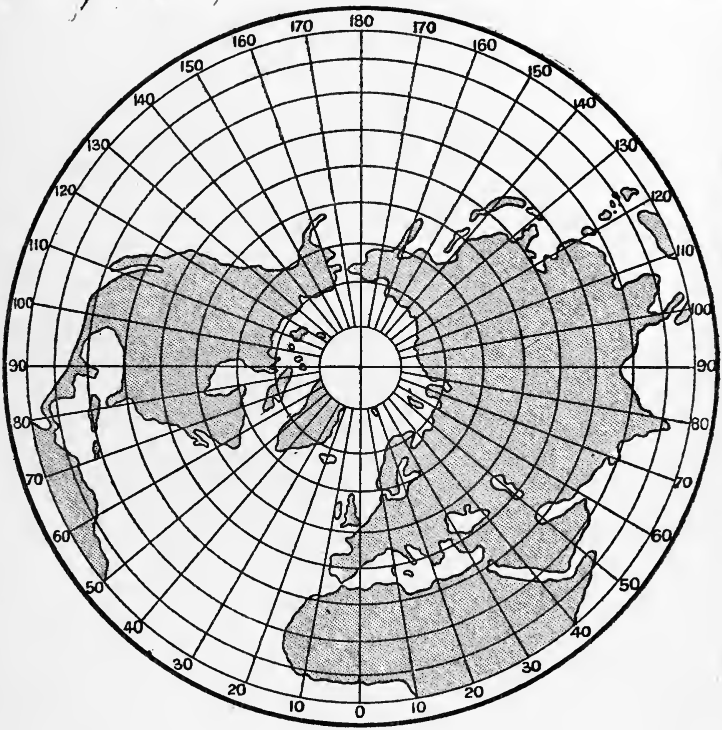
Stereographic projection FIG. 8 Hemisphere of London.




visible to the ordinary observer, but to obtain this advantage relative size is sacrificed. It is somewhat as if a ruler was represented by a billiard cue. Any short length of one would look like a short length of the other, but in a long length there is great dissimilarity.

The projection which next naturally presents itself for consideration is the PROJECTION OF EQUAL AREAS, a projection very little used, and for which, apparently, there is no special name. In the projection just above

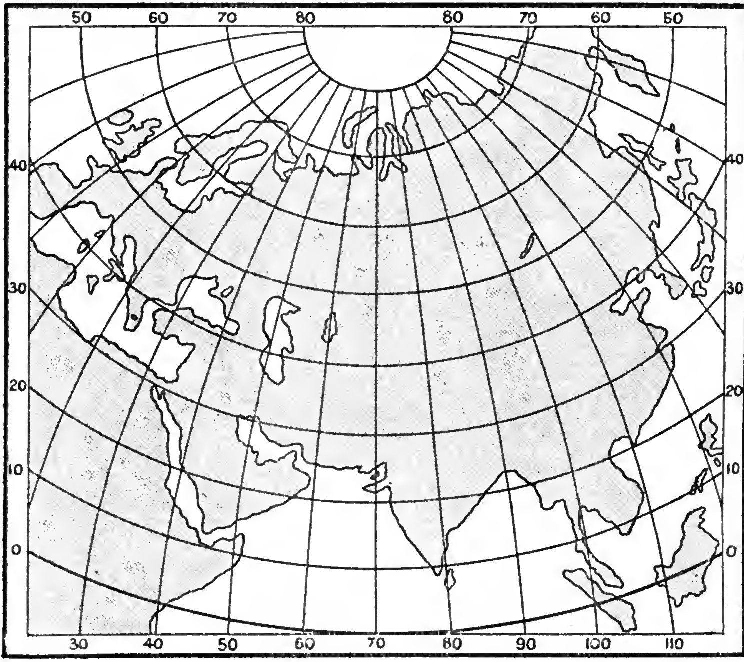
al Area Projection FIG. 9 Hemisphere – Polar




described, area is sacrificed to shape. In this projection (see figs. 9 and 9a) shape is sacrificed to area, but there is this difference, that while the shape in stereographic projection is only approximately correct, the area in this

projection is mathematically exact. Everyone knows that a piece of paper measuring 3 in. by 3 in. has the same area as a piece measuring 4 in. by $2\frac{1}{4}$ in., and it is by altering shapes somewhat in this manner that true areas are obtained. The amount of distortion, though

Equal Area Proj. FIG. 9A Europe & Asia

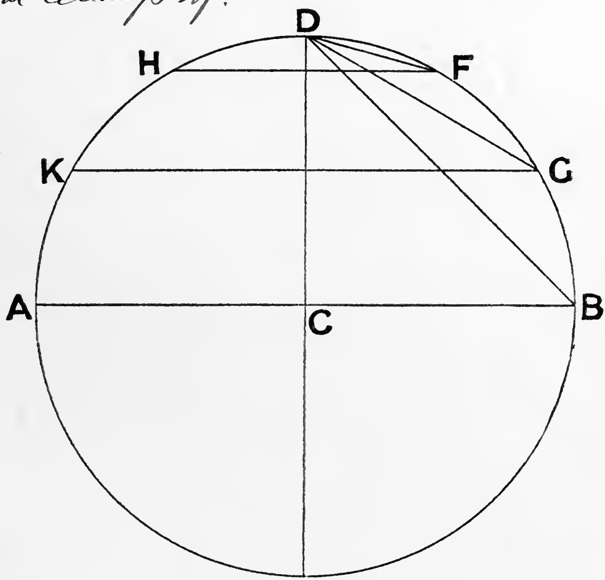



not very serious, is still sensible, as will be seen by comparing India on fig. 9 with India on Mercator's projection, where the shape of India is practically exact. But, before condemning this projection, it would be well to compare the relative sizes of India and Greenland on

this map with their relative sizes on the Mercator (fig. 13). It may well be thought that the stupendous inaccuracy of Mercator as regards size is more misleading than the inaccuracy of this projection as regards shape. In any case this map is useful, and should be more widely used, in order to give people correct ideas

Principle of Equal Area Proj.

FIG. 10

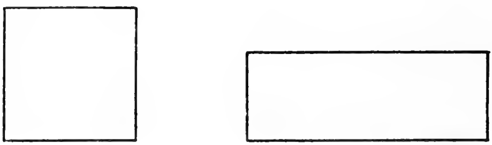



as to relative sizes, particularly seeing that, as has been stated above, the ideas of areas so obtained will be absolutely and not only approximately correct.

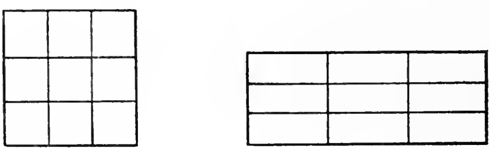
The method of constructing this projection is shown on fig. 10 Suppose on this figure D is the North Pole and A B the Equator, and H F and K G the parallels of latitude

of 60° and 30° north respectively. With a pair of compasses take the length D B as radius and describe a circle. This will be the boundary of the map and will represent the Equator. With radii D F, D G, describe other circles. These will be the parallels of 60° and 30°. Describe as many more parallels in the same way as may be necessary. Draw the meridians radiating from the centre and the projection is made.

It will of course be seen that in filling in the details a difficulty arises. Suppose lines be drawn for every degree and the correct plans of the spaces be used which were used for stereographic projection. It will no longer be possible to make them fit by simply reducing the scale. What has to be done is to fit a square into a rectangle thus:



It is clear that no alteration of scale will effect this object. What has to be done is to divide the spaces into squares and rectangles thus:



These divisions have to be made so small that a draughtsman can by eye copy into each rectangle the

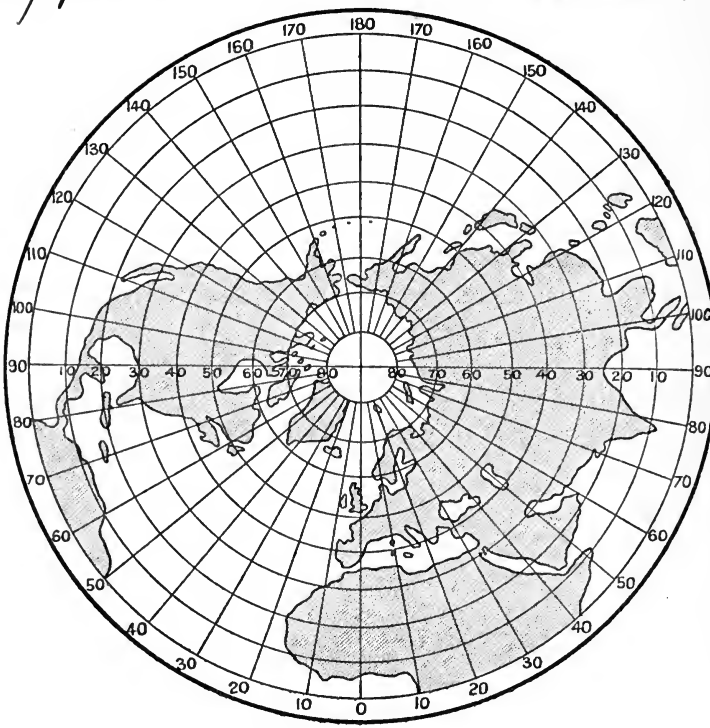
work shown in the corresponding square. The extent to which this dividing up is carried depends solely on the scale and the purpose for which the map is to be used. It is not suggested that it is carried to a very fine point in filling in the details in these diagrams, but the principle is the one universally adopted in copying from one projection to another.

Anyone who has followed this explanation so far will have noticed that on the stereographic polar projection the circles showing parallels of latitude are farther apart at the edge of the map than in the middle, while in the equal area projection the reverse is the case. This is not so easily seen on other hemispheres, but the principle there is the same, and the non-mathematical reader must accept the statement that the principles hold good throughout.

In the stereographic projection relative areas would be improved if the outer divisions were reduced in size and the inner divisions enlarged, and in the equal area projection shape would be improved if the outer divisions were enlarged and the inner ones reduced, and both these results would be obtained if the divisions between parallels of latitude were made equal throughout. It is possible to construct a projection on mathematical principles in which this will be nearly the case, but it seems much simpler to adopt equal divisions at once and call the projection the *Arbitrary Projection.* This projection is shown on figs. 11 and 12. A comparison between this and the stereographic will show that shapes are not greatly distorted, while a comparison with the equal area projection will show that areas have not been

unduly sacrificed, and it seems difficult to decide definitely against this projection. It is very easily constructed, and if children were taught the geography of the world from projections of this sort their general ideas would probably be more correct than at present.

Arbitrary projection FIG. 11 Polar Hemis ph.

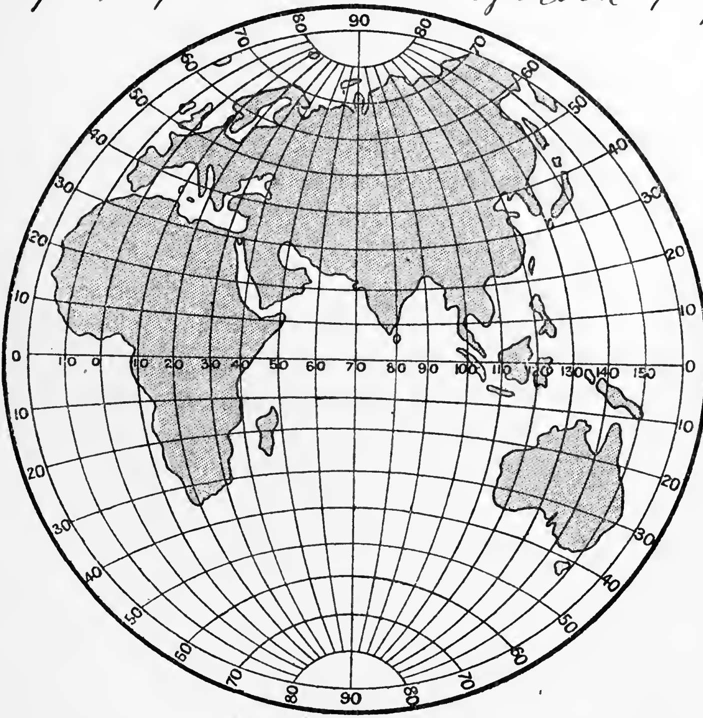



The hemispheres in the large atlas published by the Society for the Diffusion of Useful Knowledge appear to be on this projection, while those in the 'Times' Atlas are on the more generally approved stereographic projection.

A comparison of the two, in the opinion of the author, inclines one to decide in favour of the arbitrary projection over the stereographic.

All projections of the sort hitherto considered suffer

Arbitrary Proj. FIG. 12 Equatorial Proj.




from the objection which is fatal to accuracy, viz. the diameter of the circle represents a line on the surface of the earth of exactly the same length as the line represented by the half circumference.

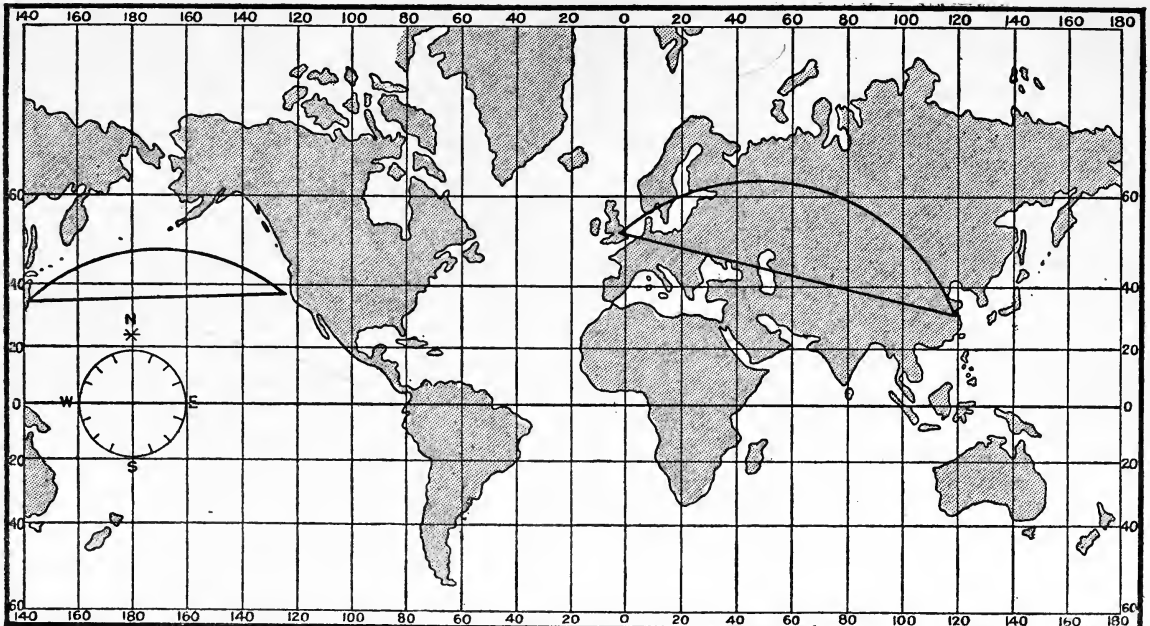
The projection next to be considered is MERCATOR'S

(see fig. 13), and here a little preliminary explanation is necessary. It has been made quite clear that a degree means the 360th part of a circle. A degree of a small circle is therefore smaller than a degree of a large circle. All parallels of latitude are smaller than the Equator, and the parallel of 60° is exactly half its size ; a degree, therefore, on that parallel is only half as long as a degree of the Equator. A portion of the earth at the Equator measuring 1° of latitude and 1° of longitude is a square each side of which measures 60 geographical miles (a geographical mile being one 60th of a degree on the meridian). A portion of the earth which measures 1° each way at latitude 60° is 60 geographical miles long measured from north to south and 30 miles measured from east to west. In any map, therefore, showing a piece of land at this latitude, if the shape is to be preserved, degrees of latitude must measure twice as much as degrees of longitude, as may be seen by looking at a map of Norway and Sweden in any atlas.

On any map, therefore, which shows a large area and on which countries at the Equator have to be shown in their proper shape and also countries at 60° latitude, either the degrees of latitude must measure the same throughout and the degrees of longitude must be proportionately reduced towards the pole, or the degrees of longitude must be kept uniform and the degrees of latitude increased in proper proportion towards the pole, or the result may be effected by somewhat altering the length of both degrees, still retaining the same proportion as they have on the earth. The third method is the one adopted in stereographic projection, the second is

*increase as the*

FIG. 13




*Mercator's Projection*

the one adopted in Mercator's projection (which projection, however, can never be made to reach the pole), and the first method is impossible on a flat surface. On Mercator's projection a line is drawn to represent the Equator and is divided into 360 equal parts. Lines are drawn square to it at every division or every tenth division, as may be thought proper. These lines represent meridians of longitude. They are parallel to each other, and consequently degrees of longitude are of the same length all over the map. The degrees of latitude are therefore increased towards the pole so that throughout they bear to the degrees of longitude the same proportion as they do on the earth. The way in which this affects the map is well seen on fig. 13. It must be remembered that the actual distance from the Equator to the parallel of 20° is the same as the actual distance from the parallel of 60° to the parallel of 80°. If the map were continued north the parallel of 89° would be as far north of the parallel of 80° as 80° is north of the Equator; that is to say, the degrees would be *on the average* about nine times as long as those between the Equator and 80°. Even at 80° one degree of latitude is six times as large as at the Equator, and consequently if there be two pieces of land of the same area on the earth, one of them at the Equator and the other at 80° latitude, the polar one on the Mercator's map is shown as having thirty-six times the area of the equatorial one. A piece of land at 89° would be shown more than 3,000 times as large as an equal-sized piece at the Equator, while a piece at the pole would be of infinite size; that is to say, the projection of the pole becomes impossible.

This excessive exaggeration of area is a most serious matter if the map be used for general purposes, and great prominence is given to this fact in this popular treatise because it is undoubtedly true that in the majority of cases people's general ideas of geography are based on Mercator's maps. As will subsequently be pointed out, this projection has many good qualities for special purposes, and for some general purposes it may be used for areas not very distant from the Equator. No suggestion is therefore made that it should be abolished, or even reduced from its position among first-class projections, but it is most strongly urged that no one should use it without recognising its defects and so guarding against being misled by false appearances.

The great feature of Mercator's projection is that the bearings of all points on the map correspond with the true compass bearings of the routes between them. For instance, on the map San Francisco lies about 2° north of east of Yokohama, and if a steamer leaves Yokohama and sails 2° north of east it will reach San Francisco. Moreover, a compass placed anywhere on the map suits for the whole map. Thus, if a ruler be laid on the route from Plymouth to ~~Panama~~ and Ber rolled carefully along to the compass set out in the Pacific, the true bearing can be read off. It was this property that first secured its popularity, a popularity which it has retained ever since, though it was known even at the time of its invention that the compass-bearing course was not the shortest route on the globe. At that time, however, long ocean voyages were uncommon. Though ships were years away from their

port of departure, and though men circumnavigated the globe, they touched frequently at ports if possible.

For moderate voyages the compass route agrees well with the shortest route, while for smaller seas, such as the English Channel, the Mercator projection is absolutely perfect, and there is this peculiarity about Mercator maps, viz. that they are all similar. That is to say, if a large number of Mercator charts of various seas covering large or small areas be reduced to such scales that their meridians of longitude are the same distance apart, they will all join up accurately so as to make a flat sheet. Each chart, that is to say, is an exact reproduction of a part of the complete chart enlarged in scale. This is not true of any other projection in general use.

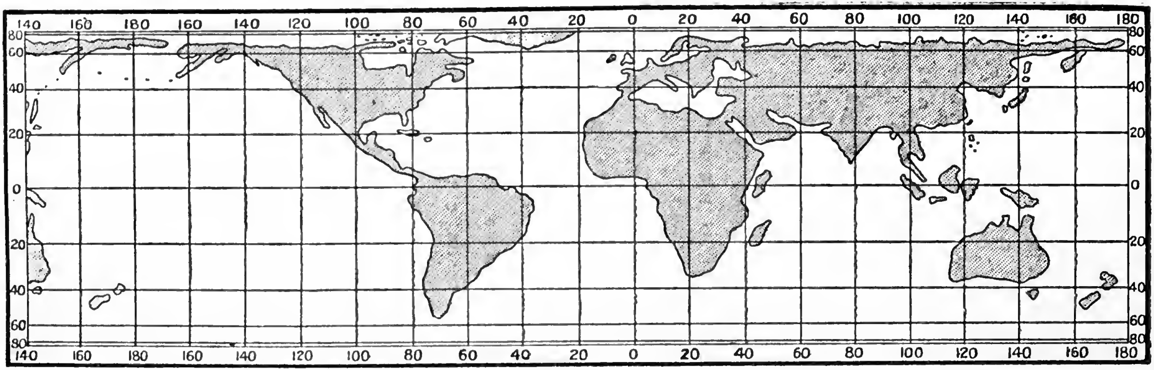
The projection also has the advantage of showing all the inhabited world on one sheet, and it can moreover be prolonged in either direction ; that is to say, it may be repeated. On other projections it is often troublesome to determine the relative positions of two places which appear on different maps which will not join, such as maps of two hemispheres. Mercator maps sometimes show the 360° of the Equator only, the usual place of division in English maps being in the Pacific Ocean. In this case the relative positions of Yokohama and San Francisco cannot easily be seen, but if a few degrees be repeated, as shown in fig. 13, this difficulty is entirely obviated. The good points of the projection then are, that compass bearings on the map agree exactly with compass routes on the globe ; that all maps, whether of large or small areas, are similar, differing

only in scale; and that the entire inhabited world can be shown on one sheet, this last property being common to several projections, though it is generally accompanied by great distortion. The bad qualities are that a straight line on the map does not even approximately indicate the shortest route on the globe, and the exaggeration in size as the pole is approached is so great as to give utterly erroneous ideas of geography in general. The extent to which polar areas are distorted is well shown by comparing the Mercator map with the map on *Cylindrical Projection*, fig. 14. This projection, which is very little known, is made by projecting each point on the sphere at right angles to the axis of the sphere out to a circumscribed cylinder touching the earth at the Equator. The surface of such a cylinder has the same area as the sphere, and the projection is an equal area projection; that is to say, all countries retain their true areas with mathematical accuracy. The distortion of shape near the poles is very great, and the projection cannot be called very generally useful; still, rightly used, it is instructive.

Having said so much on the matter of areas, the question of shortest route remains, and this can be most easily treated of when considering the next projection, viz. *Gnomonic* or *Great Circle Projection.*

The great feature of GNOMONIC or GREAT CIRCLE PROJECTION is that the shortest route on the surface of the globe between any two points is shown by a straight line on the map. The shortest route on the globe between any two points is along the great circle passing through those two points. It will be remembered that

FIG. 14



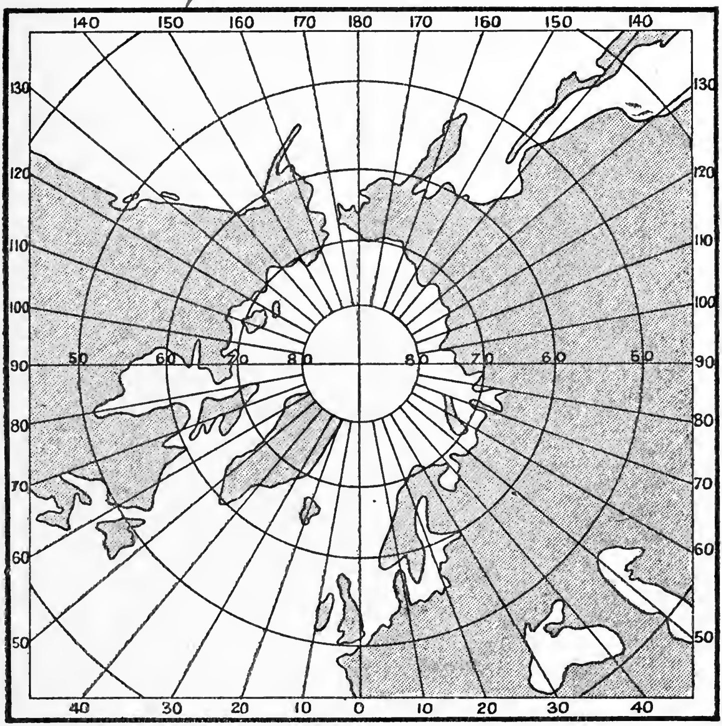

Cylindrical projection

a great circle is a circle dividing the globe into two equal portions. To illustrate this, take an orange and assuming the stalk and the mark on the opposite side to be the poles, roughly sketch the Equator and one or two parallels of latitude. Then mark two points anywhere on the orange, and examine how the orange could be cut in halves through those points. If they are on the Equator, it can be cut through the Equator. If they are on the same parallel of latitude and exactly on opposite sides, it can be cut through the poles. If they are on the same parallel, but not so far apart as the opposite sides, then the cutting line will run nearer the pole than the parallel. That the line so found is the shortest is easily tested with a piece of thread. It is always possible to cut a globe into two equal parts through any two points, so that a great circle can always be drawn through any two points, but that great circle can never run along a parallel of latitude, because a parallel is a small circle dividing the globe into two unequal parts. In Mercator's projection, if one point lies exactly east of another point, the direct route on the map appears to run along the parallel of latitude. It is true that by a long and somewhat difficult calculation the great circle can be marked out, and one such is shown on the Mercator map (fig. 13) between Yokohama and San Francisco, and tables are published to facilitate such calculations, but this part of the treatise is intended to deal with general impressions and not with navigation, and the point to be considered is this. When the ordinary business man looks at a Mercator map and is told that the route shown by the curved line from

Yokohama to San Francisco is some 200 geographical miles shorter than the route shown by the straight line, or that the curved line from London to Shanghai shows a route about 500 miles shorter than the straight one, is it easy for him to realise what this means? He may and no doubt does believe the statement, but he can hardly help thinking of the curved line as indicating a curved route in opposition to a straight one. The expression "great circle" helps to strengthen this view, and it is not uncommon to hear people talk of having taken the straight course across the Pacific (meaning the Mercator course which is sometimes taken to avoid cold weather) when in reality they have taken the curved course. Now all this goes to prove that people's ideas of geography are not founded on actual facts but on Mercator's map. Unless anyone looking at a map fairly grasps the fact that the routes above mentioned running far north are the shortest routes, unless in fact it seems to him the natural thing to talk of such routes as the straight routes, and the route shown by straight lines on the map as curved routes, his ideas of geography are badly warped, and the sooner he begins to study other projections the better. Maps on the great circle projection are shown in figs. 15, 16, 17. The principle of construction is as follows: Suppose a plane N D H K (fig. 23) to touch the globe at D. Suppose, as in stereographic projection, knitting-needles be pushed in through the points F G, but in this case directed to the centre of the earth, and suppose these to be prolonged outwards to touch the plane in H and K. By making a sufficient number of points the latitude and longitude lines can be described.

Let L M be a great circle, then the knitting-needles marking various points of it will be like spokes of a wheel, and when projected out to the plane they will

Gnomonic Proj. FIG. 15 Polar

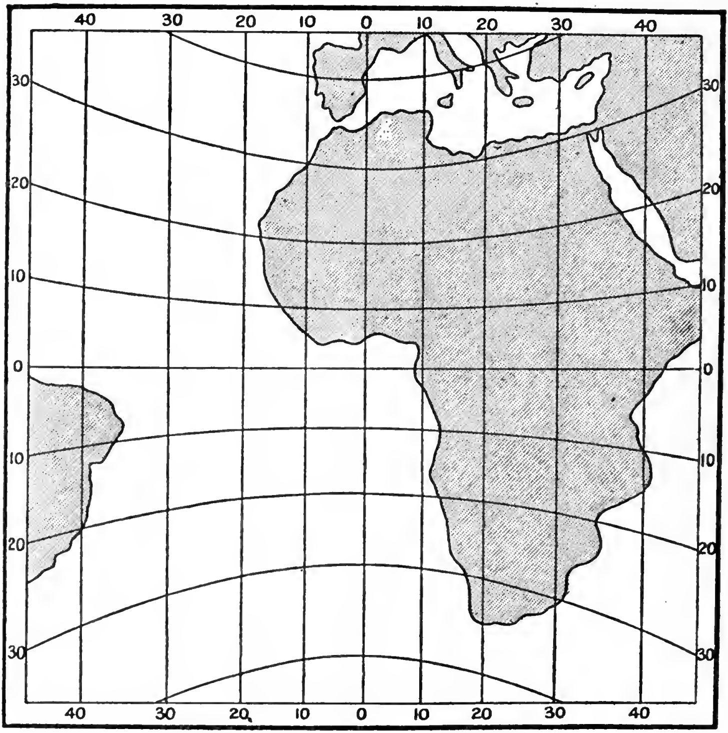



mark a straight line on it. Even to the non-mathematical mind this will become clear after a little thought.

It thus appears that on a map so constructed all great circles will be straight lines, so that the shortest route between two places can be laid down with a ruler

and a pencil. The maps are not good for general purposes, but they are useful to the general public as correctives. They are, however, extremely useful for navigation; yet, strange to say, no map on this projection

Gnomonic Projection FIG. 16 Equatorial




is published by the Hydrographic Department of the Admiralty, nor is any map on this projection to be purchased in London except one correct but badly arranged map of the North Atlantic, and British ships

sailing on the Pacific have to make use of American great circle maps.

Gnomonic Projection Indian

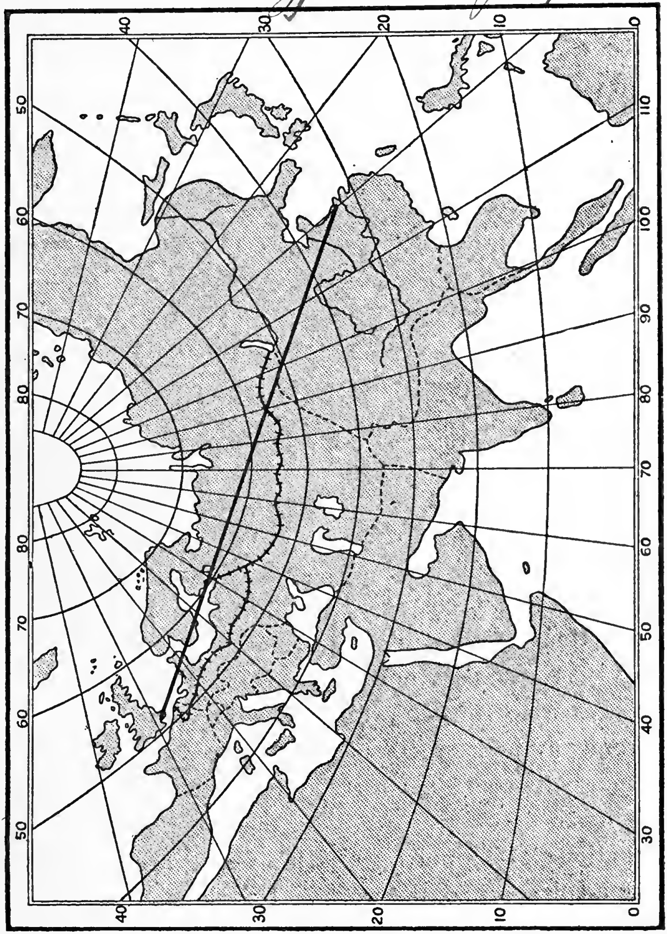



FIG. 17

The matter may be made somewhat clearer if by a stretch of the imagination the reader can conceive of

two gigantic towers being erected, one at Yokohama and one at San Francisco, with foundation at the bottom and oxygen at the top supplied by Jules Verne. If these two towers be so high that each can be seen from the other, then in imagination it would be possible to anchor buoys at intervals along the route so that they would appear to be in a straight line. Now clearly that line would be the shortest route between the places. It might be a little puzzling at first to find that the bearing of San Francisco from Yokohama was approximately north-east by east, and the bearing of Yokohama from San Francisco approximately north-west by west. Anyone sailing along the route thus buoyed out would therefore begin by sailing north-east by east, when about half-way he would find that he was sailing east, and before arrival he would be sailing south-east by east. While, moreover, he would start from latitude 35°, and arrive at latitude 38°, he would be at about latitude 47° when half-way over. A compass route would lie between the parallel of 35° and 38° the whole way, and this shows that a compass route is not the shortest route. Another way of considering compass routes is to think of what would happen if one started from any point and continued to travel north-east. It is clear he could never reach the pole, because the pole is north of every place. At the same time he would continue going north; in other words, his course would be a spiral round the world. Now a spiral of this sort cannot be a straight line, though any short piece of it may be approximately straight; consequently a compass-bearing route is not the shortest route except it be due

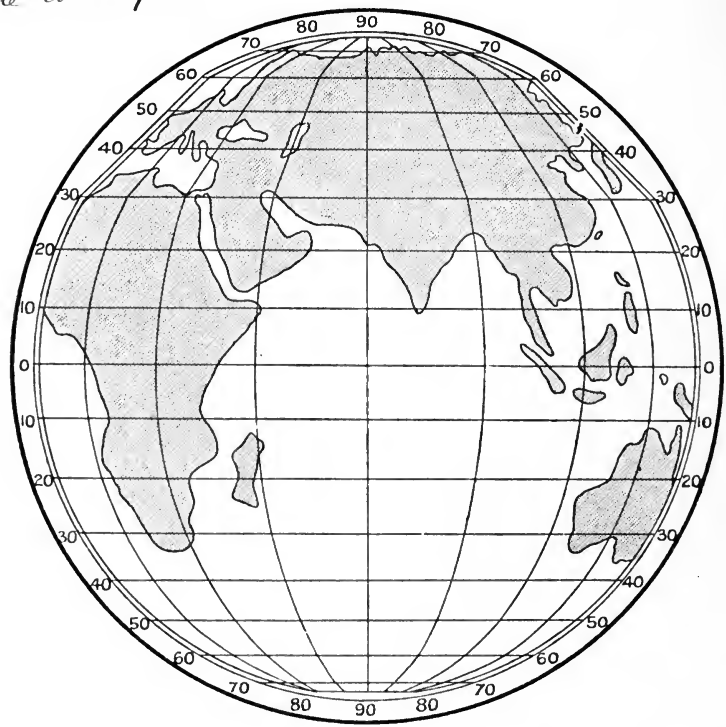
north and south, *i.e.* on a great circle called a meridian, or east and west at the Equator.

The projections hitherto described have been those suitable for showing large areas, and, as was remarked at the beginning, maps of small areas can be made so nearly correct that no distortion is apparent, and in many cases they are sufficiently correct to have a scale of miles attached. There is, however, one projection which is made use of to a very large extent in atlases for showing maps of countries which have too great an area to be shown with absolute accuracy, but still can be shown so nearly correct that the distortion is not noticeable without careful measurement. It is called *Conical* projection. Suppose a globe to be made of somewhat porous material like unglazed earthenware, and suppose the lines of latitude and longitude, as also the countries, to be painted on it, and the colour to sink in a little way, always sinking towards the centre. Then, as in fig. 18, let a band of it be turned down in a lathe, so as to form a portion of a cone. It is evident that everything that was on the surface of the sphere will appear on the surface of the cone with no appreciable distortion. Now, the surface of a cone is what is called a developable surface ; that is to say, it can be spread out flat. A piece of paper, somewhat the shape of a boy's Eton collar, can be cut so that it will lie perfectly flat on the surface of the cone, and the map can be copied exactly on to the paper ; the only distortion in the process is that between the sphere and the cone, which, even when the band covers 30° of latitude, is extremely small. The meridians of longitude are straight lines,

and the parallels of latitude are circles. Areas and distances are for all practical purposes exact, and shortest routes are very nearly shown by straight lines.

This is such a magnificent projection that it is often employed for larger areas than it is quite suitable

Sphere with partly Conical FIG. 18




for, certain modifications being introduced which generally make the meridians curved lines, but all consideration of such details must be left to the mathematical portion of this treatise.

Though the foregoing is by no means an exhaustive account of all projections which have from time to time been used, it is hoped that enough has been said to put the general reader on his guard against founding his ideas of geography on maps. The only source of true ideas is a globe, and much good would result from making it the basis of all elementary teaching, warning the scholars that maps are only tolerated for the sake of convenience, and that where map and globe do not agree the former is at fault. But before globes come into general use it will be necessary for makers to abolish that diabolical invention for misleading and puzzling people, old and young, which is still seen on most terrestrial globes, and is generally called the line of the ecliptic. It would not be one whit more ridiculous to place a picture of the sun on the meridian of Greenwich. Like the stonecutter who repaired the statue of a Knight of the Garter, and put garters on both legs, so probably at some early date an ignorant workman copied the line from a celestial globe, but how it was ever allowed to remain is an unanswerable puzzle.

Makers of globes would confer a benefit on future generations if they would make cheap globes on which was shown, not *as much as possible*, but *as little as possible.* If the oceans were shown by a light blue tint, and continents by darker tints, and if the principal great rivers and mountain chains were shown, it would be sufficient. The names of oceans and countries, and a few great cities, noted capes, &c., are all that should appear. The globe then would serve as the index to the maps of continents, which again would serve as indexes to maps

of countries. Globes as made at present are so full of detail, and are so mounted, that they are puzzling to anyone who does not understand the subject well enough to do without them, and are in most cases hindrances as much as helps to instruction.

# CHAPTER III

## PROJECTIONS CONSIDERED MATHEMATICALLY[1]

THE consideration of even the simplest question connected with the projection of the sphere involves in one sense a knowledge of solid geometry and spherical trigonometry, and the solution of all the complex questions relating to the highest class of national surveys, such as those of Great Britain or India, calls for mathematical attainments of no mean order; but almost all questions relating to the projection of a perfect sphere, and some of those relating to the projection of a spheroid, can be solved with practical accuracy by means of plane geometry, or of geometry carried such a short way into the regions of solids that it may be understood by anyone who has mastered the simpler subject. It is only assumed, therefore, that the reader has a fair knowledge of the outlines of plane geometry and plane trigonometry, and can make use of mathematical tables, such as those published in Chambers's Educational Course. The questions to be dealt with will, it is hoped, be found interesting to those who wish to know something of the subject, and many of them will, it is believed,

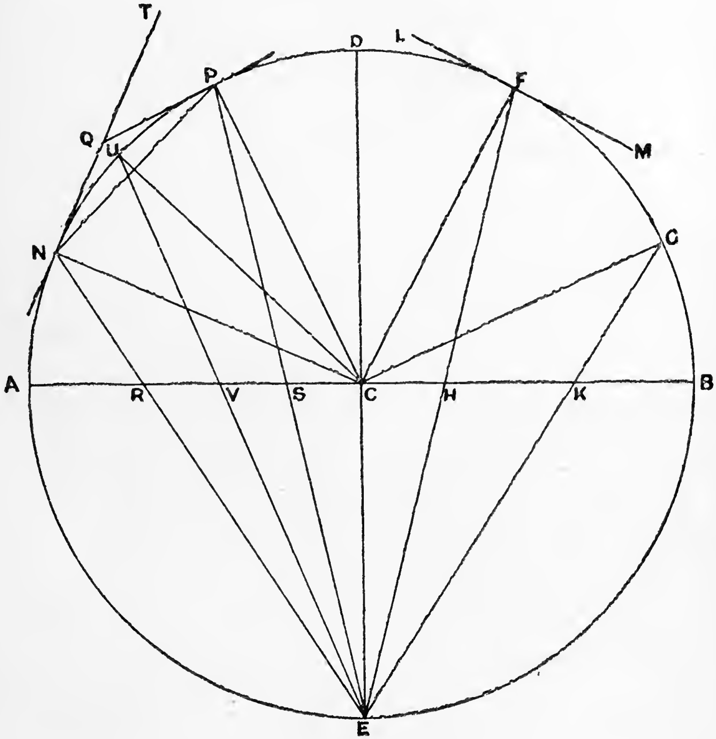
prove useful to those employed on survey or geographical work, both directly and as an introduction to the higher branches of the subject.

Following in a general way the same order as in the first part of this treatise, the first projection to be considered is *Orthographic.* A polar map is made as follows:—Draw a circle to represent the Equator, and make a scale the length of the radius, dividing it decimally. From a table of natural sines take out the sines of 1°, 2°, &c., up to 90°, and plot them from the centre to the circumference. Describe circles through the points thus found. Draw 360 radii to represent meridians one degree apart, and the work is done. See fig. 4, where, however, the lines are drawn 10° apart. To make a map showing a hemisphere intersected in the middle by the Equator, draw the circle as before, one vertical diameter to represent the central meridian, and one horizontal diameter to represent the Equator. Mark off the same divisions, viz. sines, on all four radii. Then draw lines parallel to the Equator through the marks on the meridian as in fig. 3 (where, as before, the lines are drawn 10° apart, and not 1°). The meridians are semi-ellipses, the major axis being the polar diameter, and the semi-minor axis the distance from the centre of the circle to the various marks on the Equator. Or, instead of drawing ellipses, a certain number of the parallels may have the points of intersection marked by marking off sines to a radius equal to the length of the half parallel on the map, and drawing the curves through these points. This projection, though the simplest in principle, calls for very high-class drawing.

It is practically useless for map-making, but may be employed as a stepping-stone to other projections. It has practically no good points except for a map of the moon, for which it is well suited, because, neglecting the

Principle of Stereographic projection.

FIG. 19




slight effect of libration, the same side of the moon is always turned towards the earth; it is always seen as in orthographic projection. The next projection is

*Stereographic.* In this projection the earth is supposed to be divided by a great circle, and the points on the surface to be projected on the plane of the great circle towards the opposite point. The simplest case is where the great circle is the Equator, and the point of sight one of the poles of the earth. In fig. 19, let A B represent the plane of the Equator, E the South Pole, and F G points on the surface of the sphere on a great circle in the plane of the figure. Join F E, G E, cutting A B in H and K. These points, H and K, are the projections of F and G. Now, by Euclid, angles D C F = 2 D E F, and D C G = 2 D E G; consequently arcs on the quadrant D F G B measured from D are represented on the projection by lines corresponding to tangents of half the angles measured from C. Hence, for a hemisphere of which the North Pole is the centre, draw the radii at equal angles as before, to represent meridians, and with a table of natural tangents lay off tangents of $\frac{1}{2}°$, 1°, $1\frac{1}{2}°$, 2°, &c., up to 45. Describe circles from the centre through these points, and they will represent parallels *one degree apart.* Now the effect of this is as follows: The projection of any country or network of lines immediately surrounding the pole agrees accurately in all details with the original, but is on half the scale. Again, take the case of a small area near B. On the projection a circle of radius C B represents the Equator; that is to say, any length on the Equator is represented by an equal length in the projection. Any length on the circle D F G B is represented by differences of half tangents; that is to say, 1 minute measured from B along the arc towards D is represented by the difference

between the tangents of 45° and 44° 59′ 30″ to the same radius.

Now an arc of 1 minute to radius unity measures ·0002909, while the difference of the tangents of 45° and 44° 59′ 30″ is also ·0002909; that is to say, near the Equator the scale along the meridian is the same as along the Equator. The same holds good throughout, and at every point the scale along a parallel is equal to the scale along the meridian where it crosses that parallel, and consequently every small portion of the earth is shown of its proper shape, but the scale at the centre of the map is only half the scale at the circumference. It is also clear from the figures that

$$\begin{aligned} CEH &= CFH \\ CFH + HFM &= 1 \text{ right angle} \\ CEH + CHE &= 1 \text{ right angle} \\ \therefore CHE &= HFM \\ LFH &= BHE \end{aligned}$$

Another important property must be explained. Any plane cutting a sphere makes a circle. Let a plane represented by N P cut the sphere so as to make a circle of which U is the centre, then the diameter N P will be represented by the line R S on the projection, and the lines N E, E P really represent the sides of a cone which fits the circle above described. Now this cone is not a circular cone, because its section is a circle when cut by a plane not square to its axis. It is a cone of which the cross section square to the axis is elliptical, and consequently symmetrical. Now the following relations of the angles on the plane of the figure are clear:

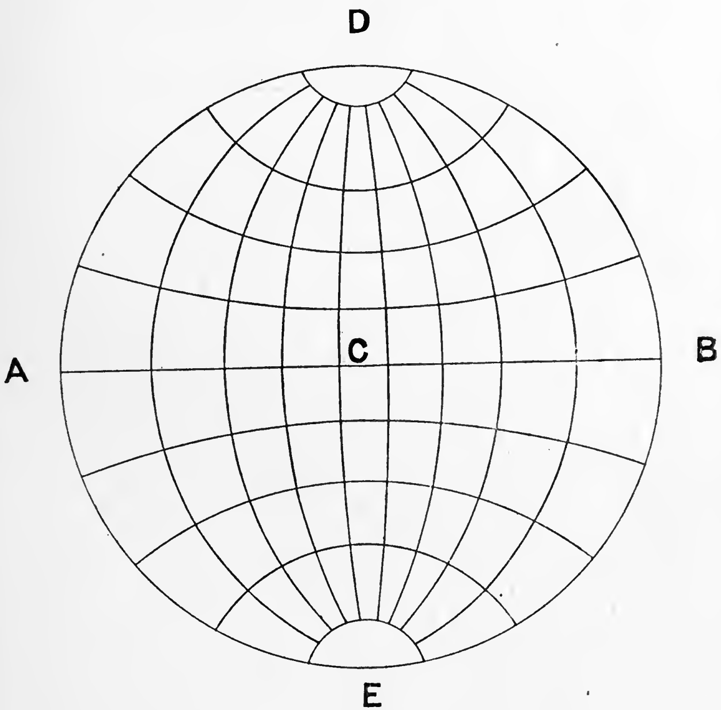
$$
\begin{array}{rl}
(1) & QNR = ARE \\
(2) & QPS = CSE \\
(3) & TQP = NCP \\
(4) & TQP = 2RES \\
(5) & TQP = 2QNP \\
(6) & PNR = QNR - QNP \\
(1)\,(4)\,(5) & PNR = ARE - RES \\
(7) & PNR = ASE
\end{array}
$$

that is to say, the plane of projection cuts the cone at the same angle as the plane of the small circle (on the other side it is true), and, as the cone is evidently symmetrical, this is the same as cutting it by a parallel plane ; consequently the projection of the circle is also a circle. It is clear, however, that the centre of the circle in the sphere is not projected into the centre of the circle on the projection, therefore this similarity of form is not of that character which would justify one in saying that all figures are represented by similar figures in the projection. They are only approximately so projected, and the approximation is only close when they are very small. The projection of the circle, however, depending, as it does, on other considerations, is mathematically perfect whatever be its size, so this general proposition is established: all circles on the sphere appear as circles on the projection, except in the special case of their passing through the point of sight, when they become straight lines (or circles of infinite radius).

This is a most valuable property, because in drawing a map on this projection it is only necessary to determine

three points in any parallel or meridian, or, in cases where the direction of one diameter of the circle is evident, only two points, viz. the ends of the diameter have to be found, and in every case the whole of the lines of

FIG. 20




latitude and longitude can be drawn with a straight edge and a pair of compasses. Having established this property the construction of an equatorial projection becomes easy. In fig. 20 let A B be the Equator and D E the poles. Draw the central meridian D E. Divide

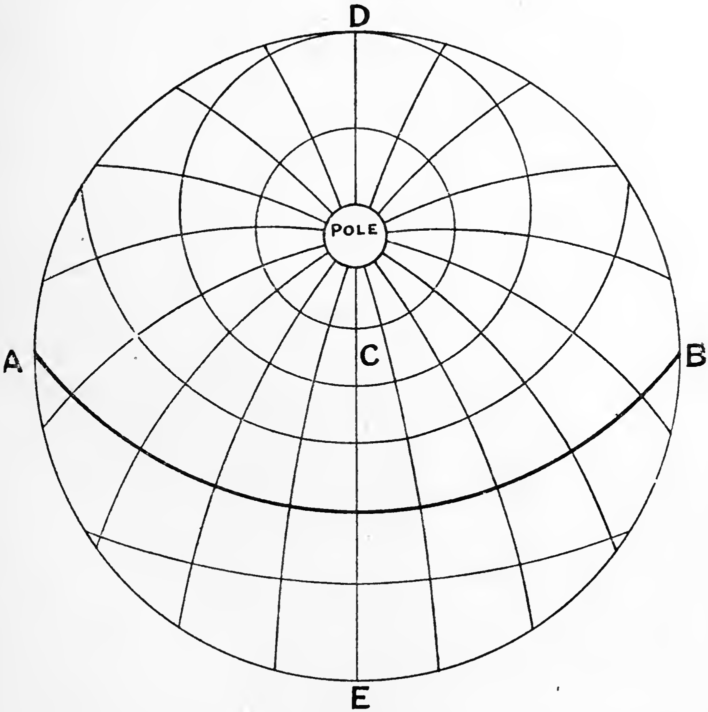
the Equator and the central meridian into ninety divisions from C outwards, corresponding to the tangents of half degrees, and divide each quadrant of the circle into degrees. Then describe *circles* passing through D and E and each marked point in the Equator, and also through each marked point on the meridian and the corresponding points on the outer circle. In the figure every twentieth line only is shown.

To make a projection of a hemisphere having some special point as centre, say the intersection of the fiftieth parallel with the meridian of Greenwich (which closely approximates with London), proceed as follows. In fig. 21, where the meridians and parallels are 20° apart, let C represent the central point and D B E A the circumference of the projection. Draw the central meridian E D and lay off a scale of tangents of half angles as before from C. The point C being 40° from the pole, the map will stretch to 50° beyond the pole; all parallels of latitude, therefore, from 40° northward will appear as complete circles. Describe circles through the two points of each parallel, the centre in each case being of course half-way between the points and not at the pole. Mark the pole P, which, being 40° from the centre point, is marked by laying down the tangent of 20°.

Then, as in fig. 22, construct a map on orthographic projection lying at angle of 50° to the cutting line G F. (The meridians need only be shown near the cutting line.) Tick off the points where the parallels of latitude cut the line G F and transfer them to the line D E in fig. 21. Rule lines at right angles to the meridian cutting the outside circle, and these points will be the points where

the parallels touch the circumference in the projection. Draw circles through these points and the tangent points first marked on the meridian. Tick off the cuttings of the meridians on the line G F in fig. 22 and

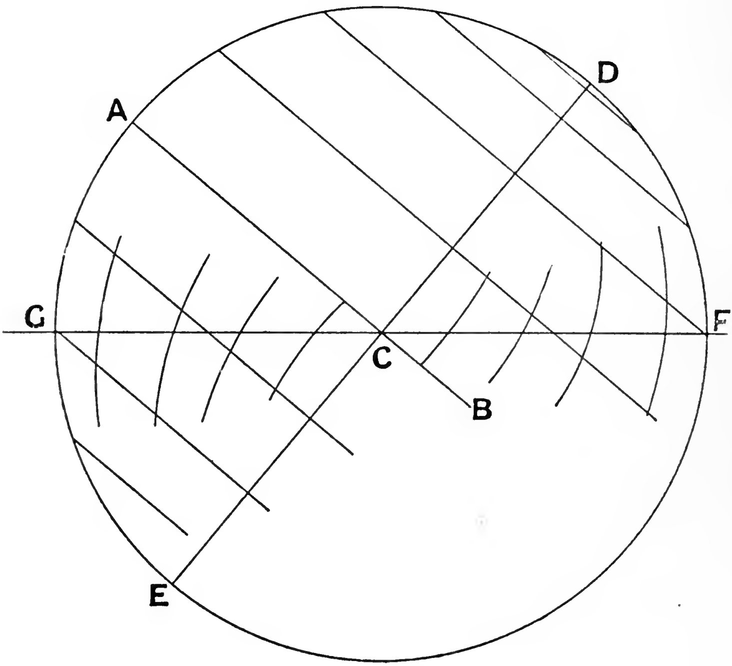
FIG. 21




transfer them in the same manner to the central meridian in fig. 21. Rule lines again at right angles to the meridian cutting the circumference. The points where these lines cut the circumference will be the points where the meridians cut it. Draw circles, as shown in fig. 21,

through these points and through the pole. This construction, though mathematically correct, will not give very good results near the central meridian except with the most accurate drawing. This may be overcome by making a polar orthographic projection and drawing on

FIG. 22




the cutting line shown in fig. 22, which will now be an ellipse, the semi-minor axis of which will be the sine of 40°. The intersections which are best shown on this diagram will be exactly those which are most un-

certain on the other. All this, however, is merely draughtsman's work.

In the *Projection of Equal Areas* the diagram (fig. 10) and the map (fig. 9) given in the first part of this treatise will serve. The circles of latitude in the polar projection are described with radii equal to the chords of the angles measured from the pole or the chords of the co-latitudes.

The circle from which the chords in the figure are taken was chosen of such a size that the chord of 90° should be equal to the radius of the circle used in stereographic projection, so that the maps should be of the same size, but clearly this is merely a matter of convenience. The surface of a sphere is equal to the surface of the circumscribed cylinder, a band of any height on the cylinder being equal to a band of the same height on the globe. That is to say, the surface of a sphere is $4\pi R^2$. Then, supposing the cylinder to surround the Equator, for any angular distance measured from the pole called A, the area of the corresponding band in the cylinder is $2\pi R^2 (1 - \cos A)$. By writing down the area of the circle of which the chord of A is the radius, and transforming it in the usual manner, it will be found that this area is also $2\pi R^2 (1 - \cos A)$. The area of the circle of which the chord is the radius is therefore equal to the area of the portion of the cylinder corresponding to that chord. All the quadrilateral figures, therefore, formed by the lines of latitude and longitude are with mathematical accuracy of the same area as the corresponding areas on the sphere.

The chord of an angle is equal to twice the sine of half the angle; therefore the projection can be made by taking a table of sines, laying off the sines of $\frac{1}{2}°$, $1°$, $1\frac{1}{2}°$, $2°$, &c., up to $45°$, describing circles through them and numbering them as complete degrees 1 to 90 (though the sines count from the pole, the numbers according to custom run from the Equator).

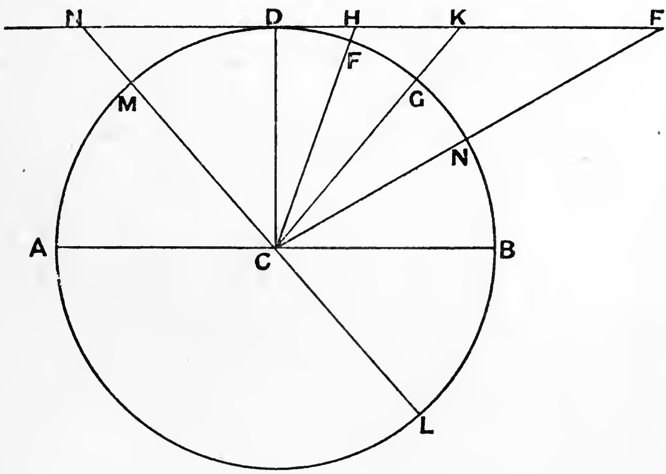
To make a projection on any other plane, make some easy projection, such as stereographic, on the required plane and place a tracing of a polar projection of the same sort over it. Then construct a polar projection of equal areas in pencil, and, noting where the lines of latitude and longitude on the stereographic projection cut the lines of its own polar projection, tick off the corresponding points on the equal area polar projection. Draw the lines of latitude and longitude through these points, rub out the pencil lines, and the work is done. This method may be used for transforming other projections, and stereographic is generally the best to take as a standard, but the equal area map (fig. 9 A) was transformed from the gnomonic map (fig. 17). The direct projection of equal areas on the plane of London is hardly possible by a direct graphic method.

Instead of following the order of the first part of the treatise, the next projection to be considered will be *Gnomonic or Great Circle* projection. In this projection all points on the surface are projected on to a plane touching the surface by lines drawn from the centre of the sphere; thus the points F G (see fig. 23) are projected on to the plane at H K.

To make a polar projection, draw the meridians

radiating at equal angles as before, and lay down a scale of tangents from the centre, as far as convenient, and describe circles through them to represent parallels of latitude. The radii of the parallels, that is to say, are the tangents of the co-latitudes. After 60° from the pole this projection is much distorted, and a projection of half the globe is of course impossible. The

FIG. 23




special property of this projection is that a straight line between any two points on the map is the projection of the great circle passing through those places, and is consequently the shortest route on the surface of the earth. To be of any real use in navigation maps must have every degree shown, every tenth degree being shown by a thicker line.

The fact that a great circle on the globe is a straight

line is easily seen when it is considered that a great circle is made by a plane passing through the centre of the sphere, and all points on the great circle are consequently projected out along that plane like spokes of a wheel, and the intersection of one plane with another plane is a straight line. These maps are useful for navigation although not very convenient sailing maps. The best way of defining and using a great circle route, or a route approximating to a great circle but avoiding extreme latitudes on account of ice &c., is to lay it down on this map and transfer it to a Mercator. No tables can be made to do this in the same time, because by this map islands, continents, ice regions, &c., can be avoided at once, whereas by tables the mathematical route must be laid down and afterwards modified. The route to avoid an extreme latitude found on these maps by drawing straight lines from the two ports touching the limiting parallel is rather a troublesome route to lay down on a Mercator.

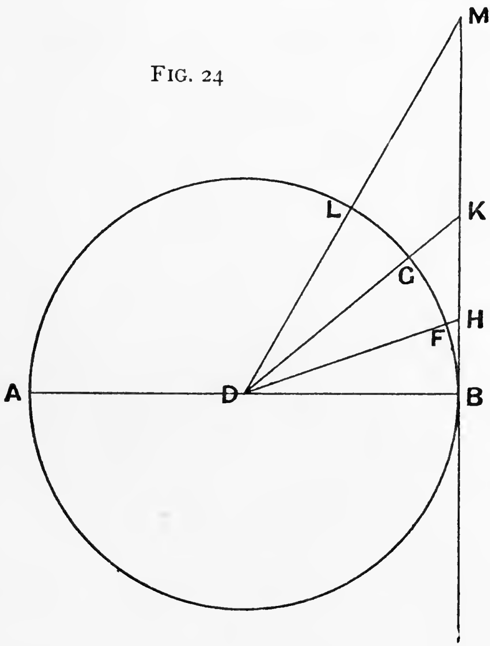
A set of maps which show the whole globe will, if fitted together, form a solid ; hence the smallest possible number of maps is four, each of which, if all are the same size, would be an equilateral triangle. It is usual, however, to have six square maps, two being polar and four equatorial maps, which form a cube. Six maps, each measuring 120° across, will, so to speak, overlap, but will form useful maps. If only 90° across they have no overlap and are not very useful.

For practical purposes, however, maps should be made to take in certain areas, such as the North Pacific, and in such cases it is often convenient to make the plane

touch at a point neither on the Equator nor on the pole, but generally at a point in 45° lat.

To make an equatorial map, let fig. 24 be a diagram looking down at the North Pole D. The circle then will show the Equator. Mark equal angles round as at F G and project them to H K. Tick off the lengths, H K, &c.,

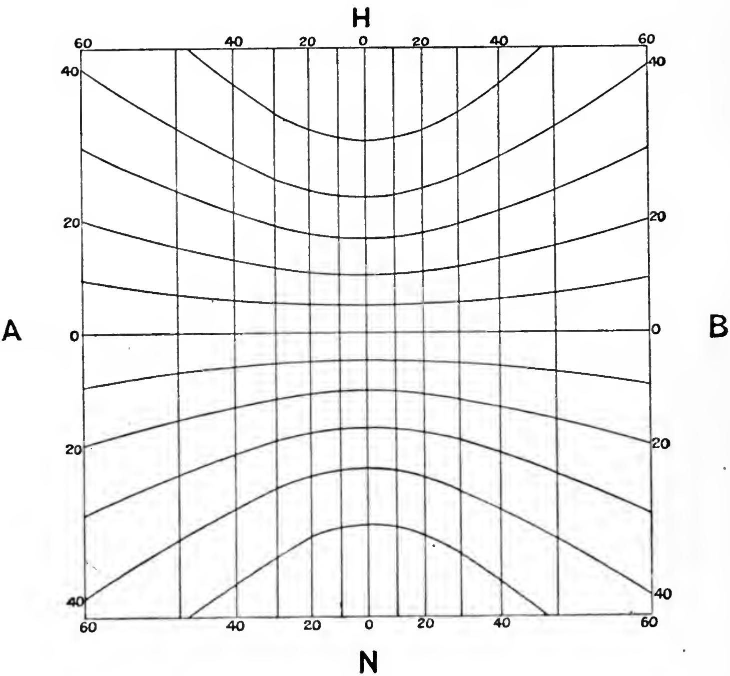
FIG. 24




and transfer them to the Equator A B on fig. 25, or, from a table of natural tangents lay down the tangent of each degree and draw the meridians through these points at right angles to the Equator. It is clear that on diagram 24, if the quadrant which is represented by the line B D were divided into degrees, and radii were drawn through

them and projected to the plane of the map, they would cut the plane at distances from the Equator equal to the natural tangents of the angles. The meridian represented by D F may be likewise divided in the same manner and

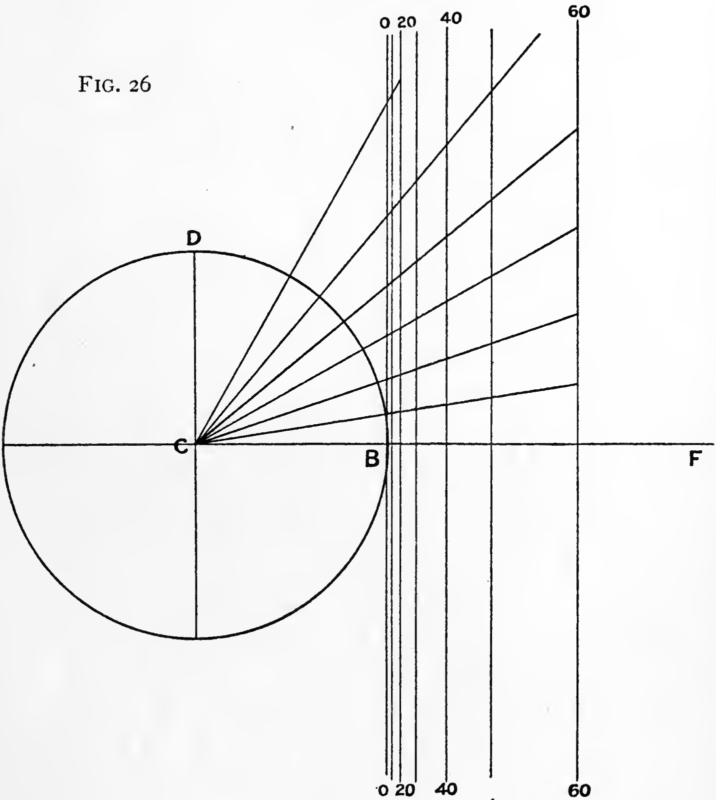
FIG. 25




the points of projection may laid down as on the meridian H N (fig. 25) also by a scale of tangents, but they will be tangents to radius D H and not to radius D F. Now D H is the secant of the angle B D H, so that on each meridian the degrees of latitude are laid down from a scale of

tangents to a radius equal to the secant of the angle of longitude between the central meridian and the meridian in question. The graphic solution is very simple.

FIG. 26




Having set out the meridians from a scale of tangents, draw the diagram (fig. 26) in which C B is equal to the radius of the sphere. Lay off, along the line C F, the natural secants to radius C B of each degree as far as it is

intended to extend the map, say to 60°. Draw vertical lines through these points, then draw radii for each degree cutting these lines. Tick off the intersections for each meridian and transfer them to the map, then draw the parallels of latitude through them. The lines will be hyperbolas, but can easily be drawn in by a good draughtsman from the points of intersection without reference to the particular form of the curve. The whole construction is exceedingly simple, but the work is considerable, as the lines must be carefully drawn at each degree and a very slight irregularity will be noticeable.

Where the plane touches at 45°, the diagram of construction becomes as in fig. 27, and the map appears as in fig. 29. The method of construction is as follows:—Consider fig. 28, which is a plan looking down on the pole and showing the map F L M K. It is clear that the line K F is shown of the same size as on the map, and the meridians cut it at distances from G equal to the natural tangents of the angles of longitude from G to radius D G. The meridians cutting the line L F as shown in plan also cut at distances from L equal to the natural tangents from L to radius D L *as they appear on the plane of the Equator in fig.* 28, but in transferring them to the map the length must be increased in the proportion of L F on fig. 28 to L F on fig. 29. All the meridians can then be drawn on the map from the point A to the points ticked off on the border line. In fig. 27 draw A G at 45° to A C and C G. Then, along C F, mark off secants of each degree to radius C G (not to radius C B). Draw lines from A to the points thus found. Draw radii from C for each degree on the quadrant D B, pro-

FIG. 27

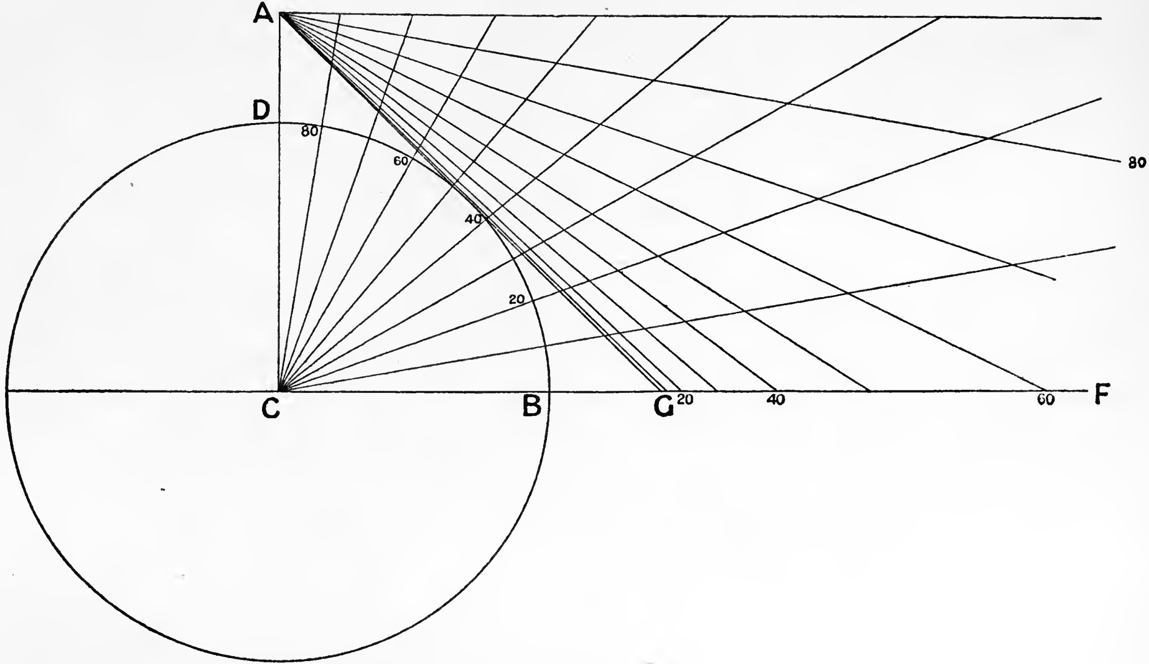
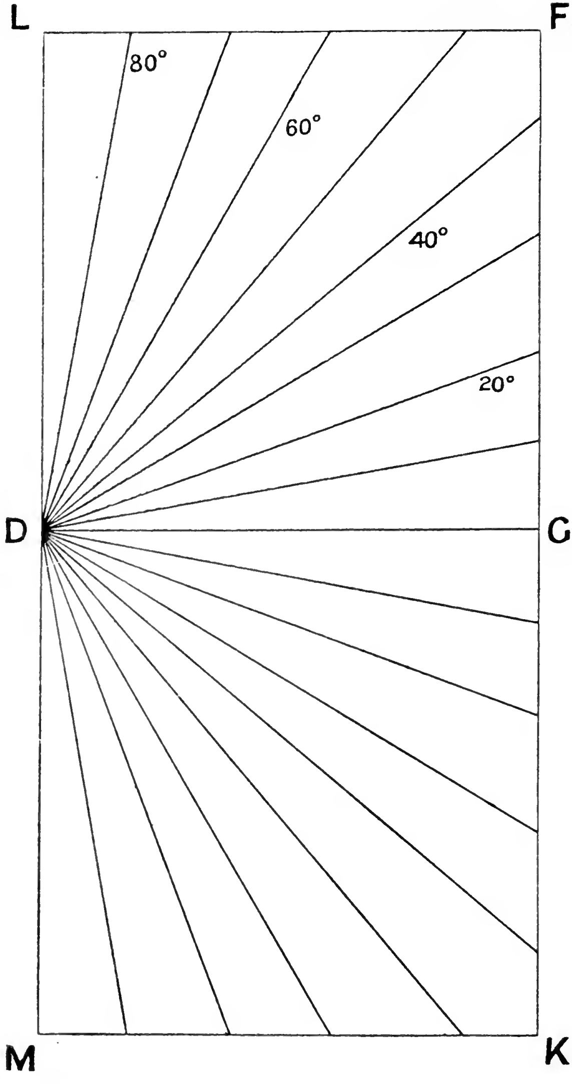


FIG. 28



ducing them to cut the lines drawn from A. Tick off the distance from A along the several meridians on slips

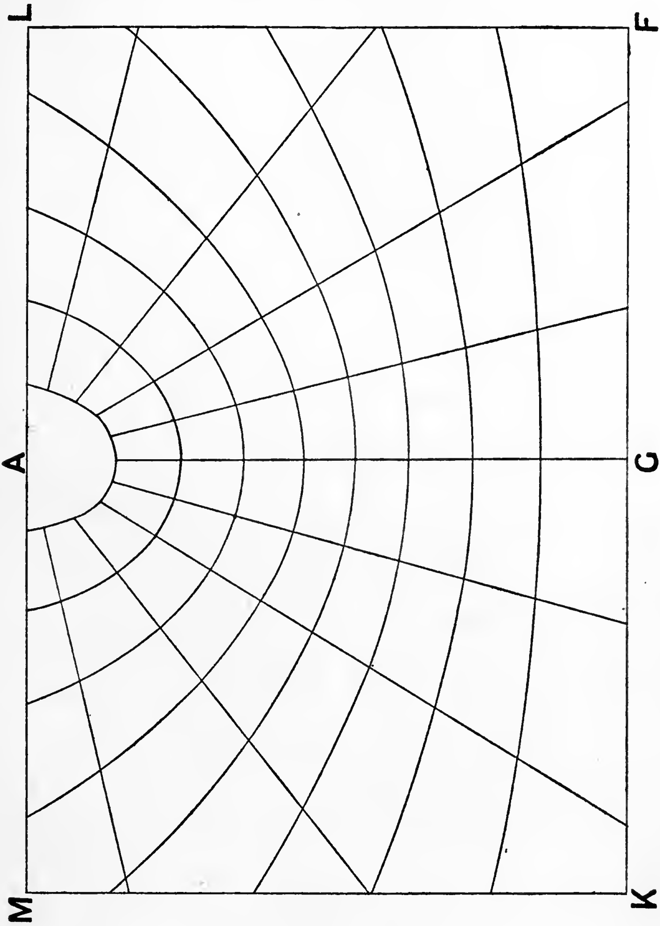



Fig. 29

of paper and transfer them to the corresponding meridians on the map. Draw curves through the points,

and the work is done. In this case all parallels north of 45° will be ellipses, 45° will be a parabola, and all south of 45° hyperbolas, but in practical construction a sufficient number of points must simply be ticked off to enable the curves to be drawn without reference to their character.

One difficulty arises out of the fact that the secant of 90° is infinity, so that one cannot plot the secant of 90° or indeed of any very large angles. This may be got over by using a table of sines at right angles to the line C F or by any of the "dodges" well known to all draughtsmen. It is believed that this simple construction of gnomonic maps has not hitherto been published. At all events it is not generally known, and indeed in many treatises but scant attention is given to this exceedingly important projection.

The four projections hitherto described may be compared by reference to the radii of the circles showing the parallels on polar projections, counting the angles from the poles or the co-latitudes.

| | |
|---|---|
| Orthographic . . | Sines of the angles. |
| Equal areas . . | Sines of half the angles. |
| Gnomonic . . . | Tangents of the angles. |
| Stereographic . . | Tangents of half the angles. |

As regards the area that can be shown:

| | |
|---|---|
| Orthographic . . | One hemisphere. |
| Equal areas . . | The whole world, but only one hemisphere good. |
| Gnomonic . . | Anything less than one hemisphere, but only 60° or 70° from centre practically useful. |

Stereographic . . Anything less than the whole world, but only one hemisphere good, though 100° or a little more from the centre may be shown without very excessive distortion.

The next projection to be considered is one of an entirely different character, viz. *Mercator's* projection. Those previously described have been capable of graphic construction, and three of them have been in a sense pictures of the globe, one from a very great distance one from a point on the surface, and one from the centre. The fourth, viz. the projection of equal areas, does not represent any view of the earth, though it looks a little like some of the perspectives. In the case of Mercator no graphic construction is possible, though it somewhat resembles a map made by projecting all points in straight lines from the centre to touch a circumscribed cylinder. The degrees of longitude on a sphere decrease from the Equator to the poles, and in order that the map of any area should agree in shape with the actual area, the proportion of length of degrees on the meridian and parallel must be kept the same as on the globe.

In Mercator's projection (see fig. 13) a straight line is taken to represent the Equator and is divided into 360 equal parts. Lines are drawn through the points of division at right angles to the Equator. These represent the meridians. Seeing, then, that the degrees of longitude on the map remain constant throughout, the only

way to keep up the proportion is to increase the size of the degrees of latitude from the Equator towards the poles. The distance of any parallel of latitude from the Equator may be found by looking up the number of meridional parts in a table of such parts in any ordinary set of mathematical tables, but no method of explaining the system on which such tables are calculated appears possible without the use of the higher mathematics. The principle, however, is easy, viz. that the degree of latitude must at every point bear its proper proportion to the degree of longitude. Thus at 60° of latitude a degree of latitude must be twice as long as a degree of longitude; but, as degrees of longitude are the same throughout, a degree of latitude at 60° must be twice as long as a degree of latitude at the Equator. A little consideration will show that the degrees of latitude on Mercator's projection vary as the secants of the latitude. Thus, if unity be taken to represent one minute as shown on Mercator, and if from a table of secants the secant of one minute be taken out, then the secant of two minutes added, then the secant of three minutes, &c., the result will be a table of meridional parts with only a slight error arising from the secant itself varying in the intervals of one minute. If one-degree intervals be taken the error will be greater; if one-second intervals be taken the error will be less.

The advantages and disadvantages of this projection are referred to fully in the first part of this treatise. It is clear that this projection can never reach the pole, but it may be repeated round the Equator to such extent as may be found useful.

There is a curious projection, very little used, called cylindrical projection. In this projection (figs. 14 and 30)

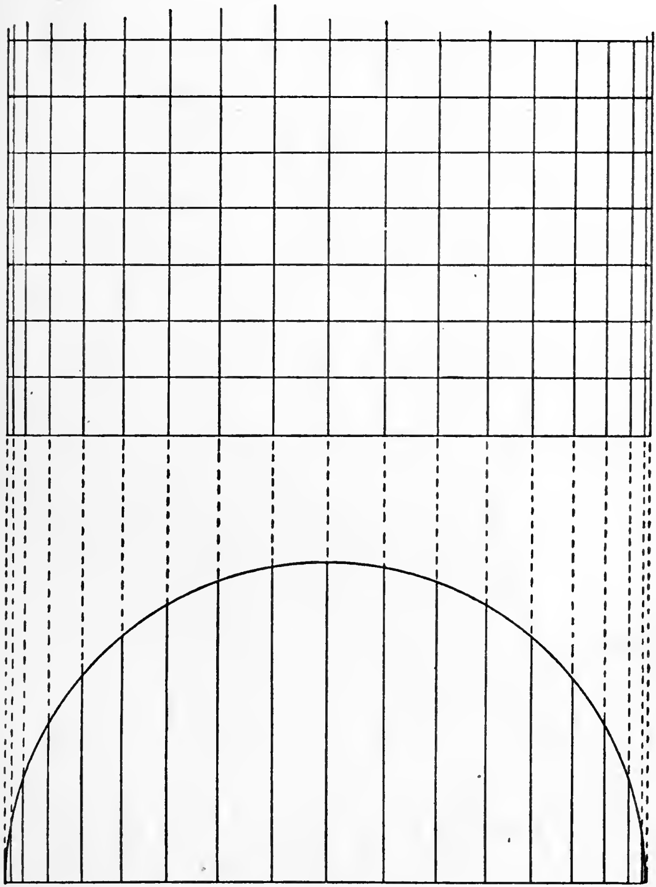
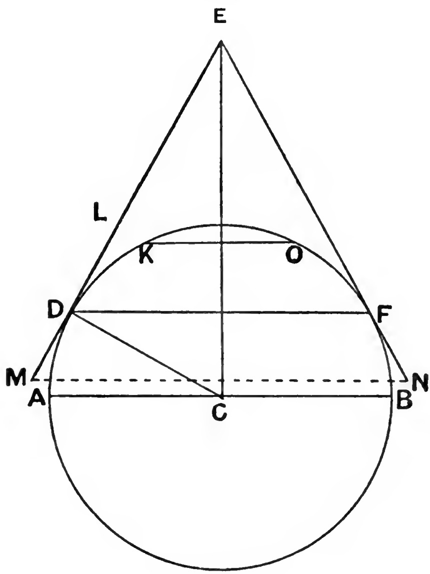


FIG. 30

the various points on the surface of the sphere are projected on to the surface of a circumscribed cylinder at right angles to the axis of the cylinder. The effect is

to give a projection of equal areas. The degrees of longitude remain the same throughout, as in Mercator; but while in Mercator the degrees of latitude are increased towards the pole, so that they may continue to maintain the same proportion as on the sphere, in cylindrical projection they are diminished in the inverse

FIG. 31

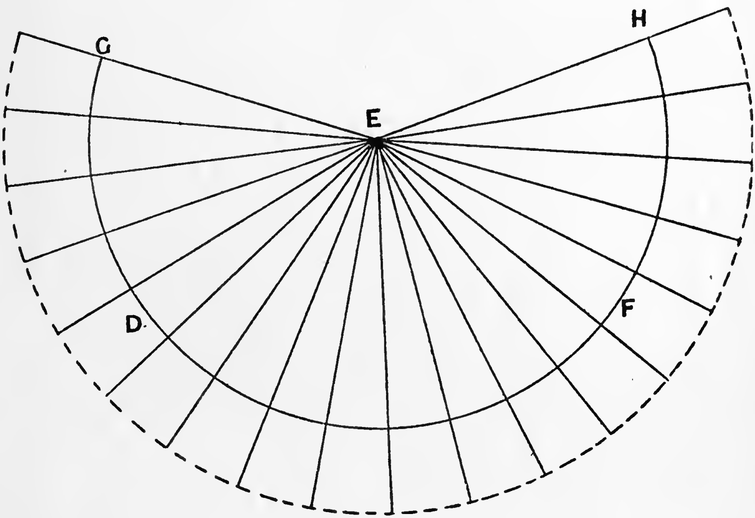



proportion, so that the rectangular figure on the map bounded by parallels and meridians agrees in area with the quadrilateral figure corresponding to it on the globe. This projection is interesting, but not directly very useful for general purposes, on account of the great distortion near the poles; but it is distinctly useful as a

corrective to Mercator, because the distortion is less than on that generally used map.

The next projection to be considered is *Conical.* It is well known that the surface of a cone is what is called a developable surface, *i.e.* it can be laid flat if cut open. In fig. 31 let the circle represent the outline of a globe of which A B is the Equator and the lines E M, E N,

FIG. 32



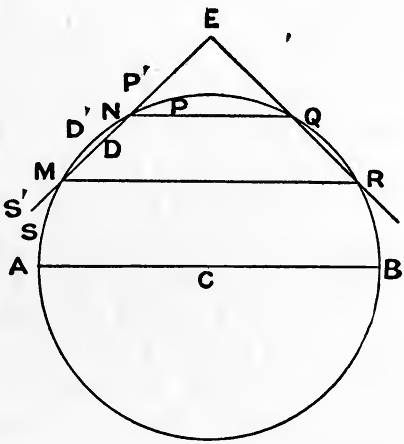

the outline of a cone touching the sphere at the parallel D F. Let fig. 32 show the conical surface laid flat, the line G D F H being the line where the surface of the cone touched the surface of the sphere along the parallel D F. This line G D F H will of course be the development of the parallel D F; and if it be divided into 360 equal parts, and radii drawn through the

points of division from E, these lines will be the developments of the meridians of longitude. So far there is no difficulty. When, however, it comes to laying down other parallels, say K O, the question arises how is the point K, fig. 31, to be transferred to the surface of the cone at L? There are three possible ways. First, K may be projected at right angles to the cone. Second, it may be projected in the direction of the radius C K produced; or, third, D L may be made equal to D K. The first plan gives an approximation of equal areas, but with distortion, because the length of the degrees of latitude, which are the same as on the sphere at the line of contact, decrease as they get farther and farther from the line, while a slight consideration of the figure will show that the degrees of longitude, which also are the same length as on the sphere at the line of contact, grow longer than on the sphere as they get farther and farther from that line. The second plan has this advantage, that the degrees of latitude increase along with the degrees of longitude, so that the proportion is very nearly the same as on the sphere, though not exactly so, but the system is little used. The third plan not only has most good points, but is applicable to modifications of the projection to a greater extent than either of the others. It is clear that, if this plan be adopted, the area contained in the quadrilateral composed of any two meridians and any two parallels will be larger on the projections than on the sphere. The projection, however, is good for a moderate area, though correct only along the parallel of contact.

A common modification of this projection is to

assume a cone which does not touch the sphere, but which cuts it in two places, as in fig. 33. In this case the parallels of latitude N Q and M R may be represented by lines on the cone of the same lengths as the parallels of the sphere. All parallels between these will be shown on the cone rather too short, and all parallels north of N Q or south of M R will appear larger on the cone than on the sphere, and by taking in a proper portion of the

FIG. 33

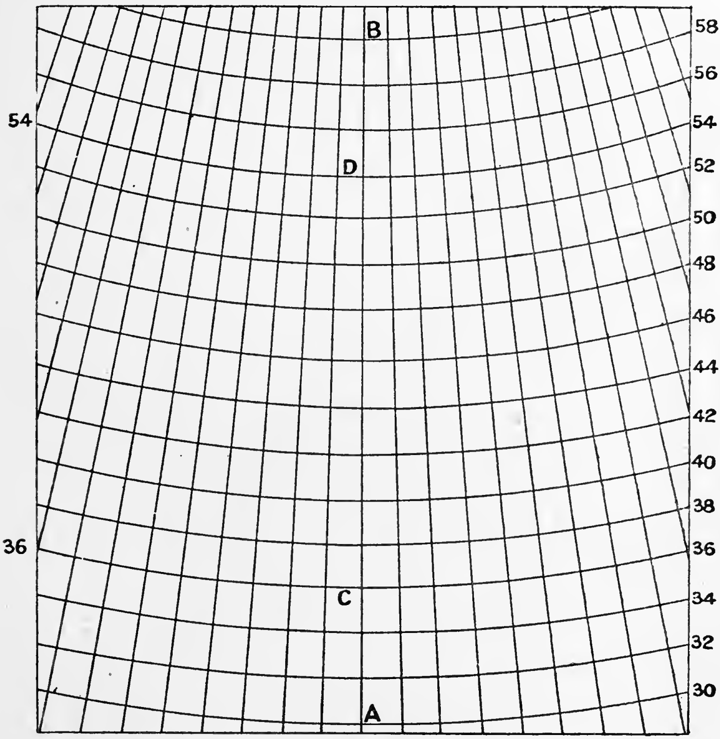



sphere the average length may be made exact, and the maximum deviation from the true length will be very small compared with that on the former system for the same area. If the three systems of projecting other parallels than N Q and M R be considered, the first system will be found to give results almost the same as with the touching cone, and the result will not be good. The second system will have some advantages,

but in common with the first it will have this anomaly, that though at M and N the degrees of longitude are the same length as on the sphere, the degrees of latitude will not agree with those on the sphere at these parallels. The third system in its direct form is impossible. If the parallels N Q and M R are to be drawn on the cone at the cutting lines, all intermediate parallels must be squeezed into a length on the cone less than the length on the sphere. Various suggestions have been made as to methods of meeting this difficulty, and the mathematical questions involved are interesting problems, but when the close approximation of the various methods is fairly considered the best plan seems to be to reject all and adopt an arbitrary method. In the first place, it is clear that, if the portion of the sphere between the lines of contact be projected on to the cone between the lines of contact, the area on the cone will be less than on the sphere. Now, portions of the sphere projected on to the cone outside the lines of contact on almost any system will have a greater area on the cone than on the sphere, and it is possible to find two parallels passing through S and P so situated that, if D′ P′ be made equal to D P and D′ S′ to D S, the quadrilateral on the cone contained between these two parallels and any two meridians will have the same area as the corresponding quadrilateral on the sphere, and the natural thing is to divide up the meridians into equal degrees of latitude between these points, but then the parallels N Q M R will not come at the cutting lines. This all points to throwing over the strictly mathematical system, and projecting as in the following

example. Suppose a portion of the globe extending from 30° to 60° north latitude has to be projected, take any line A B (fig. 34) for the central meridian and

FIG. 34

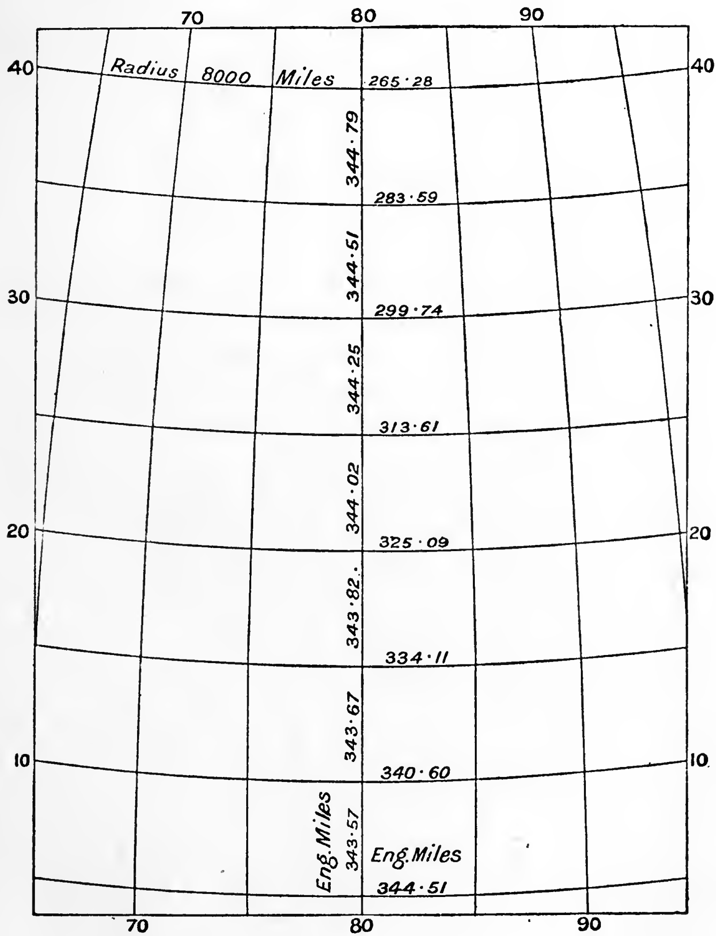



mark off 30 equal divisions to a suitable scale to represent 30° of latitude (on the figure every second degree only is shown). At 36° and 54° mark off distances to one side corresponding to the length, say, of two degrees

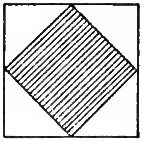
of longitude ; at these latitudes, for all practical purposes, the distances above referred to may be drawn at right angles to the line A B. Draw the line C D through these points and find graphically (or by calculation) the distance of the points of meeting. From this point as centre describe circles through the divisions of degrees. Divide off degrees of longitude along the circle passing through 36°, and draw meridians towards the centre. All degrees of latitude everywhere will be correct, and degrees of longitude will be correct at 36° and 54° of latitude. They will be too short between these parallels, and too long outside of them. If the centre of the map is very important, and the edges less important, the standard parallels may be brought in, say, to 37° or 38°, and 53° or 52°, improving the centre at the expense of the edges. This is pre-eminently a point for the map-maker and not for the mathematician. This is at once the simplest projection conceivable, and one of the best for a moderate area, and this or something like it is much used in atlases. It may be used for any range of longitude.

There is one very useful modification. Adopting the arbitrary conical projection, and choosing two suitable parallels as standard parallels, find the centre and describe the parallel circles as before, but set out degrees of longitude of the same length as on the sphere, not only along the standard parallels, but along every parallel, and draw meridians through them. The result will be as in fig. 35, the figures on which are explained in Chapter IV. In this modification there is very little discrepancy in area, but considerable distor-

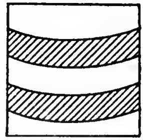
FIG. 35


70
80
90
40
40
Radius 8000 Miles
265·28
344·79
283·59
344·51
30
30
299·74
344·25
313·61
344·02
20
20
325·09
343·82
334·11
343·67
10
10
340·60
Eng. Miles
343·57
Eng. Miles
344·51
70
80
90

tion in shape at the corners of the map. In the modification described immediately before it there is very little distortion in shape or size, but what there is is distributed over a greater area. The diagrams



Curved Meridians



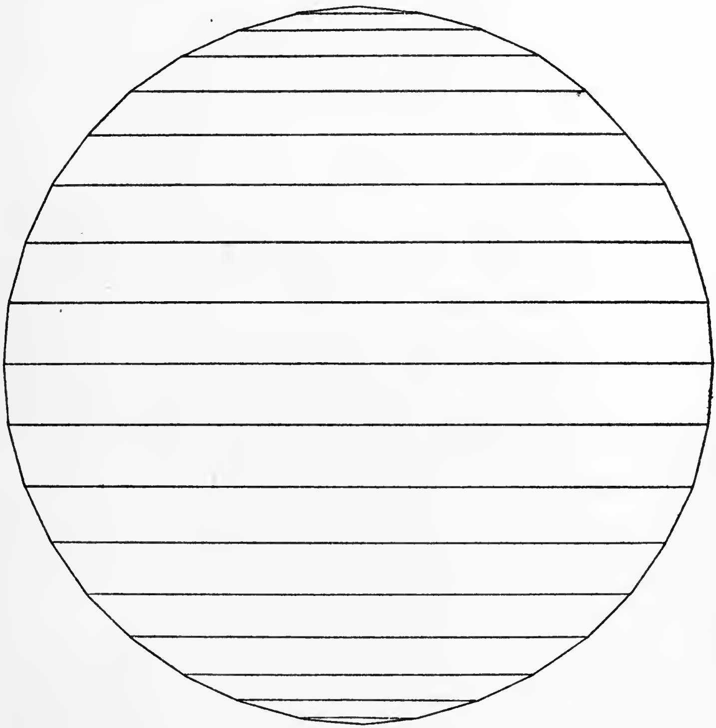
Straight Meridians

show by shading roughly the good portions of maps where these systems are adopted. The straight meridians may be best for a country like China, and the curved for a country like India.

Conical projection admits of almost endless modifications, and it is unnecessary to describe all, but there is an important one which should be mentioned. When a cone whose apex is in the prolongation of the axis of the sphere touches the sphere, the line of contact is a parallel of latitude, and on the development of the surface of the cone it becomes a circle whose radius is tangent of co-latitudes (or cotan latitude), the radius of the sphere being unity. Now, suppose the sphere to have its surface divided into a number of conical surfaces as in fig. 36, and further let the number be so great that the difference between the figure and a sphere is inappreciable, then each strip of the surface can be developed as a narrow band of equal width throughout forming part of a circle, the radius of the centre line of the band being cotan latitude, and if all the bands be laid out flat they will appear as in fig. 37.

This can be used as the basis of a very good projection, thus: Take any line (see fig. 38) to represent the central meridian of the map; assume a radius for the sphere

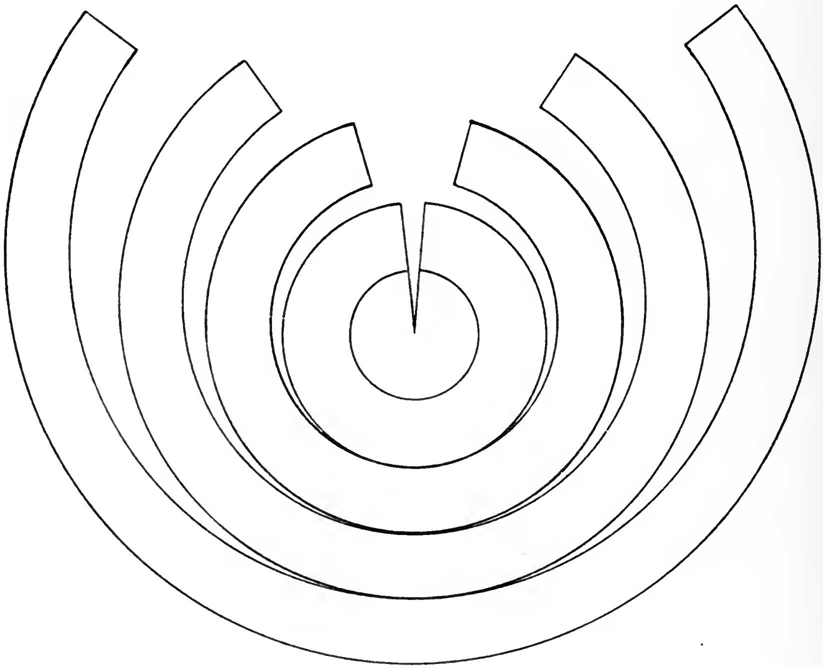
FIG. 36



and mark off equal lengths on the meridian, each equal to a degree corresponding to that radius; then through each point of division describe a circle having its centre in the central meridian and having a radius equal to

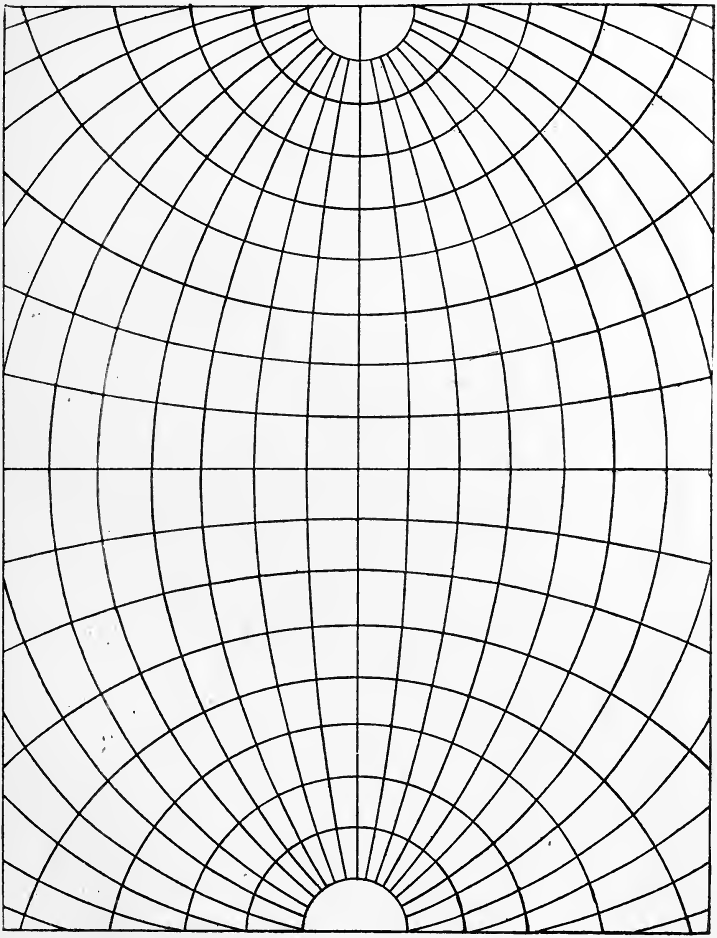
cotan latitude to the radius assumed. Mark off degrees of longitude on each parallel (in practice every fifth or tenth may be enough) and draw curved lines to represent meridians. This suits well for a map with great difference of latitude and comparatively little difference

FIG. 37



of longitude, and can be used advantageously in areas crossed by the Equator, such as Africa. In this respect it differs essentially from the simple conical projection, which is suitable only for moderate differences of latitude, but can take in great differences of longitude.

FIG. 38



There are other methods of drawing the meridians of longitude on this projection besides the one above described, but the one given appears on the whole to be the most satisfactory.

There is one point connected with projections in general to which attention may be called, though it is a point on which there may be considerable differences of opinion.

There are certain projections which have special, definite, and useful qualities, depending on the mathematical principles involved in their construction. The most important are Mercator, gnomonic, and equal area projections. In Mercator, the various places must retain the true compass bearings of the compass routes between them; in gnomonic, great circles must be shown by straight lines; and in equal area projection the countries must mathematically preserve their true areas, and to ensure this the strictest adhesion to the mathematieal principles on which they are based is absolutely necessary, and no deviation from these is permissible.

There are other projections, such as conical, which stand on a totally different basis; and there are some, such as stereographic, which may be said to hold an intermediate position. To take the latter first. Stereographic projection has a definite mathematical basis and in it degrees of latitude and longitude *at* any point bear their true relation to each other; but, as the degrees of latitude are constantly changing, the good effect of this is not shown over any considerable area. Very small portions of the earth are shown very nearly of their proper shape, and larger portions like India approxi-

mately of their proper shape; but large areas like Africa are terribly distorted. No earthly use is made of the mathematical properties of the projection. The proper way to study the shape of individual countries is to consult the maps of those countries. In a map of the world or of half the world for general use it seems more important that the relative size and position of the countries should be clearly shown, provided the distortion is not excessive, than that minute details of shape should be retained at the expense of showing one country four times as large as another when it really is of the same size. Seeing, as has been said, that absolutely no use is made of the special quality possessed by stereographic projection, there is a good deal of reason in the contention that it is more of a mathematical curiosity than a useful map projection.

There is a projection sometimes called globular, the principle of which is shown in fig. 39. The circle AFBE is the section of a sphere, and the point D is in the prolongation of the diameter AB, the length BD being $0{\cdot}707 \times$ radius.

If points GHK be taken in the circumference and lines be drawn to D, cutting CF at LMN, it will be found that the lengths CL, LM, MN are almost exactly proportional to the arcs AG, GH, HK. If then the quadrant be divided into degrees, the projections on the line CF will be almost exactly ninety equal spaces. It is clear that the half sphere can be projected on to a plane passing through EF in the same manner as is done in stereographic projection. If this be done the projection will not be distinguishable, except by careful measurement,

from the arbitrary projection shown on fig. 12, where the central meridian, the Equator, and the circumference

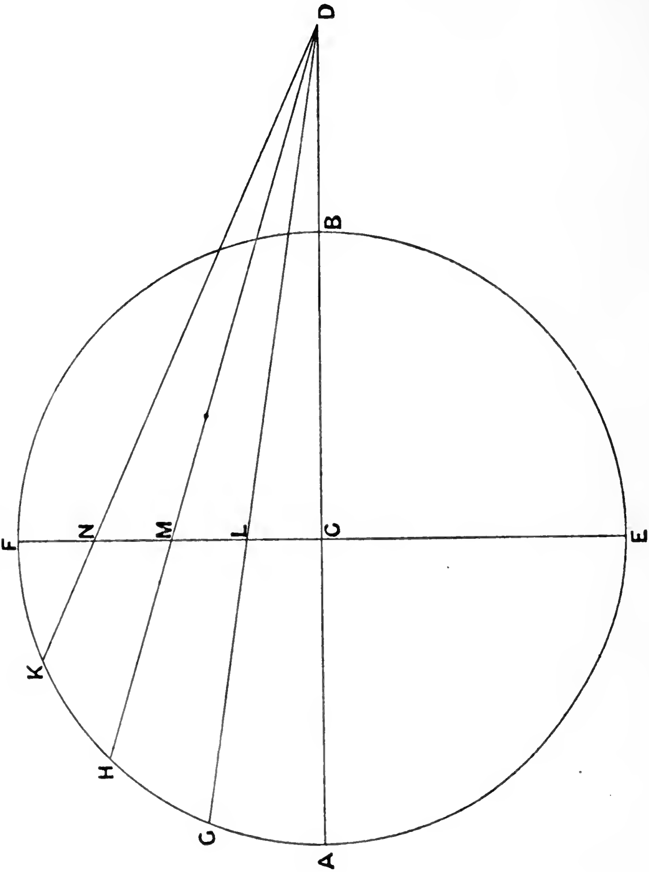



FIG. 39

are divided into equal parts, and circles described through the points thus formed. To make the projection in any other way is mere waste of labour. And it is very

doubtful if for general purposes stereographic or any other projection is better, or even as good.

In the case of conical projection, no mathematical method of projecting the sphere on to the surface of a touching or cutting cone gives such good results as the method of finding the meeting point of converging meridians and dividing the meridian into equal degrees, a system which can scarcely be said to be mathematical.

If the above views be correct, map projection is by no means merely a branch of mathematics, but an art with a mathematical basis.

# CHAPTER IV

## PROJECTIONS OF SMALL AREAS

HAVING given a general description of some of the principal projections applicable to maps of the world, and of the mathematical principles forming the basis of these projections, it may be well to touch very briefly on the question of the projection of small areas such as single countries, or even single counties, such as one sees in national surveys.

In England some town plans are issued on a very large scale, even in some cases 10 ft. to the mile (though these are no longer made at national expense), but the scales most usually dealt with are 1 in. to the mile, 6 in. to the mile, and 25·344 in. to the mile. The 6 in. maps are merely reductions of the 25 in. maps, and these maps are published in rectangular sheets, those of the 25 in. maps measuring $1\frac{1}{2}$ miles in length and 1 mile in width. The projection used is as follows: Each county or group of small counties is dealt with separately. The central meridian is taken as the co-ordinate in one direction, and a great circle cutting the central meridian at right angles about the middle of the county is taken as the co-ordinate in the other direction. The sheets are set out so as to have their bounding lines parallel to

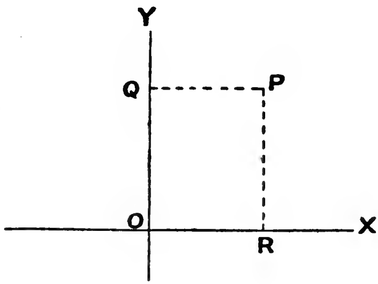
these co-ordinates, and the position of each main point is calculated with reference to them also, and so can at once be plotted in its proper place on its own sheet. It will be seen that near the centre of the county this simply means that the distance of each point north or south and east or west of the point of crossing is determined, but by degrees a difference is observable. The great circle which begins by running east and west gradually changes its bearing. A great circle, for instance, running east and west through London will cross the Caspian Sea, pass near Madras, cut Western Australia, pass a short distance south of New Zealand, cut the Isthmus of Panama, and so reach London again through the West Indies. While, therefore, it corresponds with a parallel of latitude at London, and approximately so for a short distance on each side, it cuts the Equator nearly at ~~92°~~. 52°

As the rectangular system is in use in many parts of the world, it probably deserves more than a passing allusion. No doubt a map on any projection may be divided into rectangles and the sheets sold separately, but the system here referred to is the system of plotting by rectangular co-ordinates.

On any plane surface, two lines O X and O Y (see fig. 40) may be drawn, and the position of any point P may be defined by giving the length O Q measured along O Y and the length Q P measured perpendicular to O Y, and these lengths are the rectangular co-ordinates of the point P. If P R be drawn perpendicular to O X, then the lengths O R and R P will be equal to Q P and O Q, and the position of the point P may be defined by either system of lines indifferently A system of mapping on this

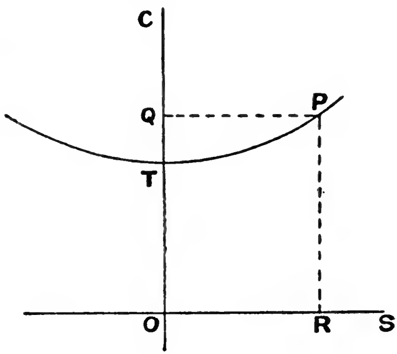
basis is employed for the Revenue maps of India, and is officially described as being based on rectangular co-ordinates computed on the assumption that the earth is a plane. This description, however, is not satisfactory.

FIG. 40




The system adopted in actual practice may be described as follows. Some great circle (usually a meridian) passing through the centre of the area to be mapped is

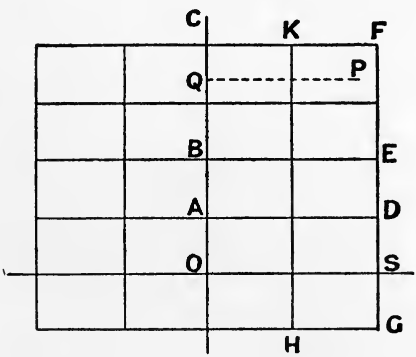
FIG. 41




chosen as the standard, such as O C (fig. 41). On this some point O, of which the latitude and longitude are known, is chosen as the origin of measurements. The position of any other point P of which the latitude and

longitude are known is fixed by drawing a great circle P Q through P to cut the standard meridian at right angles, and then calculating the lengths P Q and O Q. In practice this is usually done by means of tables from which the length T Q can be taken, T being the point where the parallel of latitude through P cuts the standard meridian. In low latitudes and for moderate distances, the length Q P may be taken as equal to T P, and in any case the difference is easily calculated; and hence the

FIG. 42




lengths O Q and Q P, which are the rectangular coordinates of P, are easily found.

A diagram of sheets is shown in fig. 42, where the line O C is the standard meridian, the lines O S, A D, B E, &c. are the top and bottom lines of sheets, and are generally considered to represent great circles cutting the standard meridian at right angles, while the lines O C, H K, G F are end lines of sheets, and all except O C correspond with small circles parallel to the standard meridian.

The point P and any number of points defined in the same way can then be plotted at once in their proper positions on their own sheets. But here a little trouble comes in. Suppose it had been decided to fix the position of P by drawing the great circle P R cutting O S at right angles, and calculating the distances O R, R P. On a plane, as has been shown, this would give the same result; but on the earth, if O Q and Q P are each 1,000 miles, then O R will be about 1,033 miles and R P about 968 miles. Therefore, the latitude and longitude of O and P being given, and great circles at right angles to each other being drawn through O, the point P may be plotted by measuring 1,000 miles (according to scale) up O C and 1,000 miles to the right at right angles, or by measuring 1,033 miles to the right along O S and 968 miles up at right angles. In other words, the earth refuses to be treated as a plane, and this projection, like all others, introduces distortion, and it only remains to be seen how far it can be employed without introducing more distortion than is allowable.

The best way probably is to select an easy case as follows: —

While in actual practice the standard great circle adopted is invariably a meridian, any great circle can be taken. If the Equator be chosen as the standard, then the subsidiary great circles are as a matter of fact meridians, and the entire system is based on a system of great circles crossing at right angles, with which everyone is so familiar that a mathematical view of the question becomes quite simple. The only difference from the usual system is that the standard great circle

runs east and west instead of north and south. On the principles stated above, if the intersection of the Greenwich meridian be taken as the origin, then the position of any other point is formed as follows. The great circle passing through this point, and cutting the Equator or standard great circle at right angles, is the meridian passing through the point. The rectangular co-ordinate in one direction is therefore the latitude of the point, and in the other direction it is the longitude *measured on the Equator.* The diagram of sheets will consist of a standard straight line to represent the Equator ; lines at right angles to it, which will represent meridians ; and lines parallel to it, which will be parallels of latitude ; and the map of the entire globe will become something between a map on Mercator's projection and one on cylindrical projection. (See figs. 13 and 14.)

All parallels and meridians will cross at right angles, and the only distortion is that arising from the fact that parallels of latitude on the earth decrease in size as they go towards the pole, or, in other words, that the subsidiary great circles converge. All lengths, therefore, measured north and south will be correct. Those measured east and west at the parallel of, say, 2° north or south will be shown too long on the map in the proportion of 0·9994 to 1. As this corresponds to a length of $\frac{1}{50}$ of an inch, in a sheet of 40 inches it is inappreciable for mapping purposes.

Now, if a belt of the earth 240 geographical miles wide can be shown without appreciable distortion on this system at the Equator, it can be shown along any other great circle, and a very good map of England can

be plotted, taking the meridian of 2° west as the standard.

But this system has one objectionable feature, which does not show itself at the Equator, nor is it of consequence when an entire map is shown on one sheet, but the only valuable quality which this projection possesses is its applicability to the issue of maps on a large scale in sheets which are all rectangular, all of the same size, and all lie flat when joined together, and it is in the extreme eastern and western sheets of maps plotted on a central meridian that this feature is seen at its worst. The objection is the one stated above, that while in the sheets at the central meridian the top and bottom lines of the sheets run east and west, this is not the case in the extreme sheets. In a map of England, as referred to above, the extreme sheets would have the bounding lines at an angle of nearly 3° from those of latitude and longitude. This would be so unsightly that to avoid it each county or group of small counties is plotted on its own meridian, with the result that the maps of adjacent counties are arranged as shown in fig. 43, and maps of adjoining counties cannot be made to join together. In England, however, where a county boundary crosses a sheet, so much of the adjacent county as is required to fill up the sheet is shown plotted on the meridian of the county to which the sheet properly belongs.

It is thus seen that the system in its only useful form is applicable only to small areas, and when extended the errors of projection accumulate until they become overwhelming and the system breaks down;

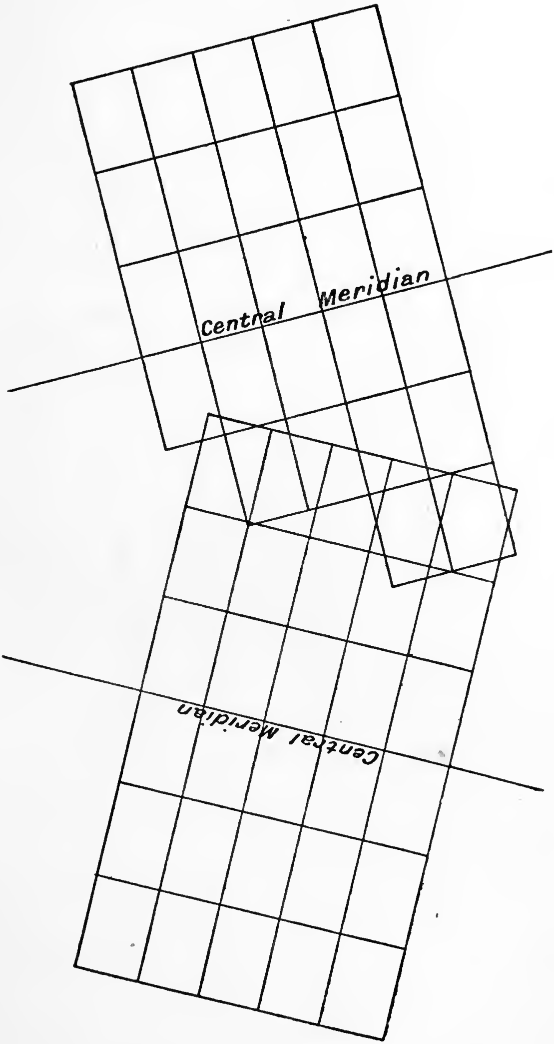



FIG. 43

while, in the system of sheets bounded by parallels and meridians, the limited and almost inappreciable distortion in each sheet is dealt with in that sheet and does not accumulate.

When a map of a large area has to be plotted to a small scale on conical or any other suitable projection, large-scale maps on rectangular co-ordinate projection are not so suitable for supplying the detail as maps bounded by meridians and parallels.

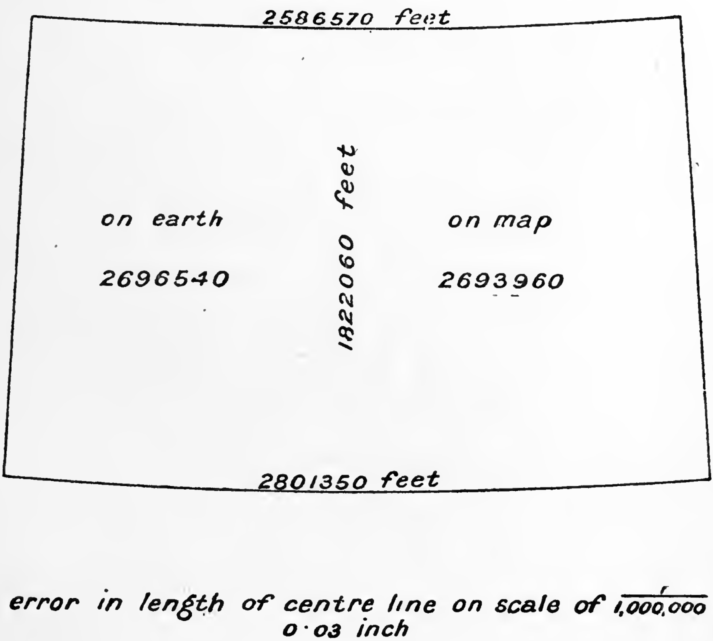
The fact, however, that the rectangular system has been adopted in England and elsewhere is a proof that, in the opinion of many people well qualified to judge, its suitability for many local purposes is so great as to more than counterbalance its great defects.

In great part of the Indian survey the sheets are bounded by meridians and parallels of latitude, and are consequently of the shape shown in fig. 44. The bounding lines are generally laid down by a system of diagonals, but in reality the centre line of each sheet north and south is its central meridian, and the centre line east and west is a circle whose radius is cotan. latitude (subject to a slight correction on account of the spheroidal form of the earth), the top and bottom lines being parallel to it, and the end lines being the end meridians and being straight.

The advantage of this system is that it is universal. A certain number of sheets can be joined together, say four, or sixteen, or sixty-four, according to the scale, and this applies to any part of the map, even if it be a map of Europe, or, for that part of it, of the world, while in the other system sheets plotted on the same meridian

(and necessarily within a limited area) can be joined perfectly, and sheets plotted on different meridians not at all. Further, the latitude and longitude of any point being known, it can be plotted at once on its sheet irrespective of scale, or of the system to which it belongs,

FIG. 44




and the small-scale map forms an index map to all the other issues.

The disadvantages are that the maps do not join *exactly*, even for an area such as that of an English county, that the shape is not pleasing, and that they are

not all of the same size. These latter objections, however, are trifling. In the opinion of the author the balance of advantages is distinctly in favour of the sheets bounded by meridians and parallels.

The survey of an area such as a country, as everyone knows, is founded on a system of triangulation. Anyone who can follow this treatise knows how, on a plane surface, the length of one line being given, the distance and direction, of various points can be determined. The spher~~oidal~~ical form of the earth introduces difficulties. For instance, if B appears to be north-east from A, then A will not appear to be south-west from B. Further than this, the earth is not a perfect sphere, but spheroid, and all calculations have to be made with reference to the curvature at the place of observation. But here again another difficulty comes in. If one knew the exact form of the earth it would be hard enough to make the necessary calculations, but the exact form is not yet known, and the very observations of the survey may have to be used to determine the form. How an approximate form is arrived at and is used to form the basis of further observations, and how these observations are further used to improve one's knowledge of the form, are matters lying far outside the limits of this treatise; but in a general way it can be understood how, by measuring a base line and carefully fixing the latitude and longitude of its ends, the latitude and longitude of a large number of points situated roughly some thirty miles apart can be determined by triangulation, a few points having their latitude and longitude fixed again by astronomical observation as

checks. Sights are often taken over much greater distances than thirty miles, sometimes five times as much ; but, roughly speaking, the difficulty attending longer observations introduces more chance of error than is got rid of by using long sights. This is called the primary triangulation. After this is finished the large triangles are cut up into small triangles, and this is called secondary triangulation. Any further work is done by theodolite and chain, or by plane table or otherwise, but in all cases on the basis of surveying a plane, as spherical errors are inappreciable.

The main points of the survey having been plotted on to their respective sheets, and then the points of the secondary triangulation, the detail is filled in as above mentioned by surveyors, the lettering put on, parish or county boundaries shown, and the plans generally finished up, contours or hill shading being added according to scale.

If, as is generally the case in the best surveys, the large-scale plans are finished first, the next work is to reduce them to the next smaller scale by photography or otherwise, so as to make sheets containing four, or sixteen, or sixty-four sheets, as the case may be.

The difficulties of reduction are of two distinct kinds. First, there is the question of the amount of detail to be shown, the size of the lettering, &c.. This, it may be remarked in passing, is often got over by taking an impression of the large-scale map in light blue colour, then drawing over the portion to be reproduced in thick black lines, putting in large dark lettering where necessary, and then reducing by photography. The

black work only comes out, and not the light blue. This, however, has nothing to do with the principles of reduction. The second difficulty is that a time comes when the reduced sheets refuse to fit together even approximately, so as to lie flat. This occurs when a map on a small scale covering a large area has to be produced. In that case the projection to be adopted for the small-scale map has to be settled, the lines of latitude and longitude drawn on, and the position of each sheet marked off on the projection. The large-scale sheet must then be reduced by other means than photography. It may be divided into small quadrilaterals, bounded by parallels and meridians, and corresponding quadrilaterals be drawn on the general map, and the sheet reproduced by a draughtsman; but the essential point is that the general projection must be laid down, the parallels and meridians drawn so as to form quadrilaterals, each holding a sheet, and the sheet must be forced into its place. Whenever the area to be shown is so large that there is sensible distortion, any attempt to build up the map from sheets, instead of laying down the projection of the general map and forcing each sheet into its place, is sure to end badly.

The exact point at which difficulties come in cannot be distinctly specified, but with sheets bounded by meridians and parallels it may be said to begin when the meridians forming the side lines cannot well be shown by straight lines.

Before proceeding further it is necessary to explain distinctly what is meant by latitude. In general terms, the latitude of a place is the angle which the plumb-line

at the place makes with the plane of the Equator measured along the meridian. This is equal to the altitude of the *celestial pole in the heavens*, and is found by observing that pole directly, or else observing the altitude, when on the meridian, of one of the heavenly bodies whose polar distance is known. It may, however, happen that there is a large mountain range in the neighbourhood which deflects the plumb-line and so makes the angle of the plumb-line with the Equator different from the angles at places due east and west of it. As this local attraction affects water level and of course all spirit levels, it is extremely difficult of detection, but it appears to amount to about one minute of arc in the neighbourhood of the Himalayas. The latitude deduced from observations of heavenly bodies is properly called the astronomical latitude of the place, and when corrected for local attraction it is called the geographical latitude. As a rule, however, the two are treated as identical, and they will be so treated in this treatise.

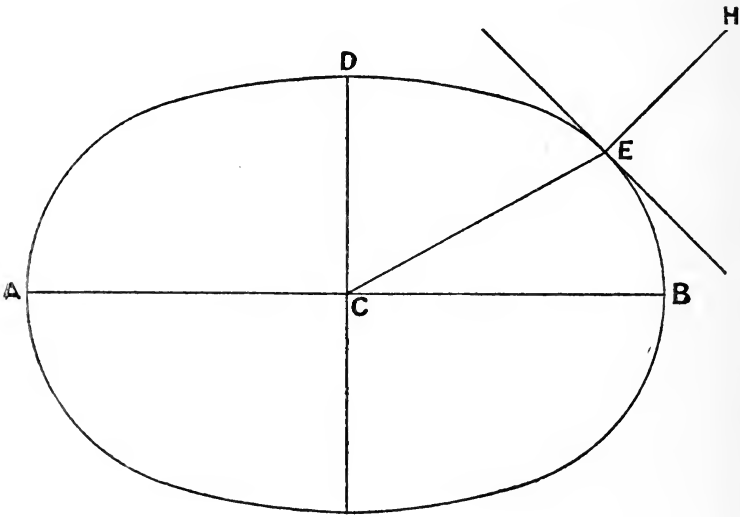
Let fig. 45 be a section of the earth through the poles (the spheroidal form being exaggerated), and let A B be the Equator, and D the North Pole. Let E be a point where the vertical line E H is at 45° to the equatorial diameter, and the tangential or horizontal line at the point E likewise at 45° to the same diameter. The geographical latitude of E is then 45° north.

Geocentric latitude, however, by which is meant the angle E C B, is essentially different. Assuming the line C B to measure 20,923,600 ft. and C D 20,853,660 ft.,[1] then the

[1] I have adopted these figures for two reasons: partly because they are adopted by Chauvenet and Loomis, and very convenient tables are based

angle ECB is nearly 44° 48′ 29″, and the line CE about 20,888,800 ft. and the length of a degree of longitude at geographical latitude 45° is $\frac{20{,}888{,}800 \cos 44° \; 48' \; 29''}{57 \cdot 29578}$ (the denominator being the degrees in arc = radius) This comes out 258,658 ft., a degree of latitude being 364,572 ft. In the same manner the geocentric latitude

FIG. 45




corresponding to 40° geographical latitude is 39° 48′ 40″, and to 50° geographical latitude it is 49° 48′ 40″, while the radius in these cases is about 20,894,850 and 20,882,700 respectively. This radius must not be confused with the 'radius of curvature' of the meridian, a radius which

upon them, and partly because they agree with the figures given in Chambers's 'Logarithms,' which was mentioned as a convenient book to be used in connection with this treatise. Clarke gives 20,926,202 and 20,854,895.

clearly is shortest at the Equator and longest at the poles. These figures give the means of calculating the length of a degree of longitude at 40° and at 50°, and testing how far that at 45° differs from the mean of these two. But the figures show more: they show that the delineation with accuracy of a considerable area on a large scale is a matter of considerable difficulty, involving a knowledge of mathematics somewhat greater than is possessed by the ordinarily educated man. It also shows that it is futile to go into details unless this is done with a care and minuteness quite unsuited to an elementary treatise supposed to be capable of being read by anyone with a knowledge of plane geometry and trigonometry. For the sake, however, of those who wish to follow all the calculations as far as they can, an example will be given of the calculations necessary for determining the amount of distortion on a sheet of a certain size on the assumption that the earth is a perfect sphere of radius 20,889,000 ft., and then a table will be given of the lengths of degrees required to lay down the parallels and meridians for one sheet of a map of the world and for a map of India, taking into account the spheroidal form of the earth.

In the case of a sphere a degree of longitude at any latitude is equal to a degree at the Equator multiplied by the cosine of the latitude. On this basis the following table is calculated :—

| Latitude | Length of degrees of Longitude |
|---|---|
| 40 | 279280 feet |
| $42\frac{1}{2}$ | 268794 ,, |
| 45 | 257797 ,, |
| $47\frac{1}{2}$ | 246304 ,, |
| 50 | 234345 ,, |

The mean of the length of degrees at 40° and 50° is 256812 ft., while the mean of those of $42\frac{1}{2}°$ and $47\frac{1}{2}$ is 257549 ft. If each of these be subtracted from 257797 (the true length of a degree at 45°), the remainder is 985 in one case and 248 in the other, being 0·38 and 0·097 per cent. of the length of the degree respectively, showing that if a sheet were made to take in 10 degrees of latitude and were bounded by straight side-lines to represent meridians and curved lines of the proper length to represent the northern and southern parallels, then the centre parallel would be 0·38 per cent. too short. This is a larger error than is allowable. If, on the other hand, the sheet covered 5° of latitude, the error would be only 0·097 per cent., or say 5 ft. per mile, and this could be made even less by making the northern and southern parallels slightly too long. A map covering 5° or 300 geographical miles of latitude would be presumably on a small scale, and the following statement may be accepted. If for any purpose lines be laid down on a sheet to represent an area of 5° of latitude and perhaps 10° of longitude with straight side-lines as meridians, and the top and bottom lines be circles drawn from a centre distant co-tangent latitude from the centre of the sheet, and if that sheet be divided into quadrilaterals corresponding to sheets of a large-scale survey, each sheet of the large-scale survey can be fitted into its own place by simple reduction to scale, the difference of shape or size between the reduced sheet and its corresponding quadrilaterals being inappreciable; and this will hold good whether the earth be treated as a sphere or a spheroid, the sheet in either case being to all appearance like fig. 44.

This has a distinct bearing on the question of the best projection for a map of the world on a scale of $\frac{1}{1000000}$ as proposed at the International Geographical Congress.

In the first place it is clear that the map must be in sheets. Now a sheet taking in about 5° of latitude and say 10° of longitude at 40° latitude would measure about 33 inches by 22 inches, a very convenient size. It has been seen that such a sheet can be plotted with parallels and straight meridians as bounding lines without sensible distortion, and if the views hitherto expressed have anything in them at all, there can be no doubt that the map should be plotted in sheets bounded by parallels and meridians, each sheet being plotted on its own central meridian and taking in 5° of latitude and say 8° of longitude at the Equator, then 10°, then 12°, &c.



It would be necessary for the Committee of the Congress to fix the exact form of the earth to be adopted and to issue tables giving the length of each degree of latitude, and also the length of each degree of longitude at every 5° of latitude, and the corners of the sheets could be fixed by giving the radius of curvature of each parallel, or by giving rectangular co-ordinates from the central meridian and a line at right angles to it at the bottom of the sheet, or by giving diagonals from the central meridian. If these tables were issued and the standard meridian fixed (probably Greenwich) the work of laying down outlines could begin at once.

Sheets on a large scale could be set out so that they would exactly fit into these sheets, while for maps of continents the sheets would supply the detail after the

general projection suitable to each case had been laid down.

Fig. 44 shows a sheet extending from 40° to 45° latitude and taking in 10° of longitude. The following are the measurements, assuming the earth to be a spheroid of dimensions previously given :

| | |
|---|---|
| Meridian 40° to 45° . . | 1,822,060 ft. |
| Parallel 10° long. at 45° . . | 2,586,570 ,, |
| ,, ,, 40° . . | 2,801,350 ,, |

As the convergence of the meridians is 214,780 ft., they will meet at a distance of about 21,942,800 ft. On the scale mentioned above, therefore, the parallel of 45° will have a radius of 22 ft., or more exactly 21·94 ft. For international work, no doubt, measurements would be given in metres, but the above indicates a workable size of sheet, and finding the radius by the convergence of meridians is much better than the method usually given, viz. : Assume a cone cutting the spheroid at 41° 44° and ~~40°~~, and find the apex by calculating the normal of each parallel from the axis of the earth and deducing the radius of the parallel on the development from the length measured along the cone from the apex to the cutting line. It is true the difference is not great, and the calculations are no doubt simple, but they are not so simple as those given above, and any difference is distinctly in favour of the method here adopted.

The question of the best projection to be adopted for the general map of a large country is one requiring a good deal of consideration, and anyone having to decide it would, among other things, probably lay down one or two projections on a small scale and study them

The following notes on the principal points connected therewith may interest the reader, though they are of too sketchy a description to be of use to anyone who has made a study of the subject. Suppose a map of India be required on a large scale, say 16 miles to the inch. This would be a large wall-map, probably the largest map that could conveniently be used. The properties of conical projection have already been considered, and the modified conical with curved meridians has been indicated as probably the most suitable for such a map. The first thing to be done is to make a table of the lengths of degrees of latitude and longitude, taking into account the spheroidal form of the earth. These must be taken from tables, just as one takes sines and cosines. The calculations of such tables may be looked upon as belonging to the very highest branch of the subject. Those readers, however, who are using this as an elementary text-book may like to be able to make calculations for other places as exercises, and may not have access to tables. The following approximate formulæ are sufficiently correct for this purpose :

$$\text{Length of a degree of latitude in feet at latitude } \phi = 364570 - 1820 \cos 2\phi$$

$$\text{Length of a degree of longitude in feet at latitude } \phi = 365492 \cos \phi - 307 \cos 3\phi$$

Most published tables give single degrees in feet, but the following condensed table gives lengths of 5° in English statute miles as being more suitable for the present treatise :

| 5 degrees of Latitude | | 5 degrees of Longitude | |
|---|---|---|---|
| Latitude | Length in miles | Latitude | Length in miles |
| 5 to 10 | 343·57 | 5 | 344·51 |
| 10 ,, 15 | 343·67 | 10 | 340·60 |
| 15 ,, 20 | 343·82 | 15 | 334·11 |
| 20 ,, 25 | 344·02 | 20 | 325·09 |
| 25 ,, 30 | 344·25 | 25 | 313·61 |
| 30 ,, 35 | 344·51 | 30 | 299·74 |
| 35 ,, 40 | 344·79 | 35 | 283·59 |
| | | 40 | 265·28 |

One point which will strike the reader who has not been accustomed to deal with these matters is that the degree of longitude at 5° latitude is larger than the degree of latitude.

Having made this table, set out a straight line as central meridian and on any convenient scale mark off the degrees of latitude. Then mark off the degrees of longitude at each degree of latitude on one side at right angles to the central meridian, and draw a line through the points thus found. This line will be slightly curved. Take a piece of thread and move it so as to average the curve by eye, and draw a fine line and get the appearance of it well into mind. Then rub it out, marking the two ends for future reference. Then consider the facts. The portion south of latitude 10° contains only the point of India and Ceylon. Seeing that the meridians are to be curved, the point to be chosen as centre for the parallels will not affect the projection of these places in the slightest degree. The portion of the parallel will to all appearance be straight in any case, and the degree of longitude will be of the correct length. This portion may therefore be omitted from consideration.

Seeing that the 5° at the south of the map may be left to take care of themselves, attention must be turned to the north. While the northern area is not so important as the central, still it stretches over a considerable number of degrees of longitude and therefore must be considered. Take the thread again and arrange the line between 10° and 40°. Marking this on the diagram and turning again to the map, it will be seen that the latitude of Calcutta is about 22½° and of Bombay about 19°, and that the original line, averaging the entire length from 5° to 40°, will give a projection suiting Calcutta and Bombay better than the second line.

Having got so far, it will be well to come to figures. The first line will probably cross the parallel of 40° about 275 miles from the central meridian, and will probably be inclined to the central meridian at an angle of about 1 in 30·4. It will consequently cut the central meridian at a distance of about 8,360 miles.

The second line will probably cut the parallel of 40° at a distance of about 271 miles from the central meridian and will be inclined to it about 1 in 27·4, and will consequently cut the central meridian at a distance of about 7,430 miles. In these cases the word 'about' is used because no two people will choose exactly the same line. The difference at first sight seems enormous, but consider the real effect. The width of the map will be about 30° of longitude, or say 1,591·7 miles on the parallel of 40°. If the circle to represent this be struck with a radius of 7,430 miles, the arc will be an arc of 12° 16′, and the versed sine of the arc will be 42·5 miles. If it be struck with a radius of 8,360 miles the arc will be an

arc of $10^\circ\ 54\frac{1}{2}'$ and the versed sine will be 37·8 miles, a difference of 4·7 miles. Now this does not represent an error: it simply means that on a map 8 ft. wide the parallels in one case will have about $\frac{1}{4}$ of an inch more curvature than in the other.

It thus becomes clear that the fixing of the position of the centre of the circles of latitude in the modified conical projection with curved meridians is a much less important matter than in the conical with straight meridians, as it then affects the angle of the meridians and the projection generally, while with curved meridians it may safely be said that within tolerably wide limits it is of no consequence.

This may be stated in a different way. Suppose it has been determined to make a map of India stretching from 10° to 40° of latitude, and the projection has been fixed. Suppose, then, that it be determined to extend it south to 5°, the question will arise: must a new projection be laid down, or should the old projection be retained and the piece at the bottom added? The question will resolve itself simply into a question of the distance of the centre of the circles forming parallels, and it should be answered not altogether on mathematical considerations, but on the basis of the importance of the centre and edges of the map. It is possible, no doubt, to work out mathematically the exact amount of distortion at any part of the map under each of the conditions, but no use is ever made of such a calculation, as the projection has no mathematical qualities. The two radii mentioned above are probably the extremes which might be chosen, and, if a radius of 8,000 miles

be adopted, the projection will be as good as any other. This statement may not commend itself to everyone, but it is doubtless true if the statement formerly made be accepted, viz. that map projection is not a branch of mathematics, but an art with a mathematical basis.

When this has been settled the projection is practically complete. Take the paper on which the map is to be made, and lay down the central meridian (see fig. 35). Lay off the points of crossing of the parallels. Describe a circle of radius 8,000 miles (according to scale) to represent the parallel of 40°, and circles from the same centre for the other parallels. Along each circle mark off the degrees of longitude of their correct length, draw curved lines through these, and the work is done.

As regards filling in, it is clear that a sheet on a scale of sixteen miles to the inch, plotted to its own meridian, and showing an area at the centre of the map, will fit practically into its place. A sheet so plotted, but showing an area near one corner, will not fit, but must be transferred by the method of drawing corresponding quadrilaterals on the two maps, but the distortion in the worst case will be remarkably slight. It is evident that while the projection above described is excellent for India, and is indeed probably the best possible, it would be unsuitable for a much greater length of longitude. In such a case straight meridians must be adopted.

It is hoped that the preceding pages will serve the purpose intended, viz. will enable anyone without mathematical knowledge to understand various pro-

jections in a general way, and recognise the advantages and defects of each, and likewise will enable anyone with a fair knowledge of elementary mathematics to understand the mathematical basis of the various projections referred to.

It is likewise hoped that the treatise may be useful both as a text-book to young students who do not require to go deeply into the subject, and as an elementary text-book to those whose profession requires them to master map projection in all its details, and to whom the more advanced text-books might at first be rather puzzling.